X(3872) Particle Electromagnetic Decay Properties

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Introduction

Belle(3003): $B^{\pm} \to K^{\pm} J/\psi \pi^{+}\pi^{-}$, Later $X \to J\psi\pi^{+}\pi^{-}\pi^{0}$

CDF and D0: $p\bar{p}$ collision

BaBar: B decay

M=3871.56$\pm$0.22 MeV $\approx$ M($D^0 \bar{D}^{*0}$)

$\Gamma < 2.3$ MeV at 90% C.L.

$X(3872) \to J/\psi\gamma$ confirms the positive C-parity.

Quantum number: 1^{++} or 2^{-+}.

? Large isospin broken and small mass

Models to explain X(3872):

Charmonium [We use this assumption (Phys. Lett. B 697 (2011) 233)]

Diquark-Diquark [Maiani, Piccinini, Polosa]

Hybrid [Li]

Molecule [Swanson, Close, Voloshin, Wong, Liu, Zhu, Dong, Braaten]

Cusp effect or a virtual state [Bugg, Hanhart, Kalashnikova, Swanson]

BS equation

BS equation has the following form

$$(\not{p}_1 - m_1)\chi(\not{p}_2 + m_2) = i\int \frac{d^4k}{(2\pi)^4} V(p,k,q)\chi(k). \quad (1)$$

$$\begin{aligned} p_1 &= \alpha_1 p + q, \quad \alpha_1 = m_1/(m_1+m_2), \\ p_2 &= \alpha_2 p - q, \quad \alpha 2 = m_2/(m_1+m_2). \end{aligned} \quad (2)$$

Instantaneous BS equation has the simple kernel,

$$V(p,k,q) \Rightarrow V(k,q) = V(|\vec{k}|,|\vec{q}|,\cos\theta). \quad (3)$$

where

$$q^\mu = q_\parallel^\mu + q_\perp^\mu, \quad q_\parallel^\mu \equiv (p\cdot q/M^2)p^\mu, \quad q_\perp^\mu \equiv q^\mu - q_\parallel^\mu. \quad (4)$$

In the mass center frame, we have $q_\perp = (0,\vec{q})$.

We introduce

$$\varphi_p(q_\perp^\mu) \equiv i\int \frac{dq_p}{2\pi}\chi(q_\parallel^\mu, q_\perp^\mu) \tag{5}$$

$$\eta(q_\perp^\mu) \equiv \int \frac{\vec{k}^2 d|\vec{k}| d\cos\theta}{(2\pi)^2} V(\vec{k}, \vec{q}, \cos\theta)\varphi_p(\vec{k}). \tag{6}$$

$$\Lambda_{ip}^{\pm}(q_\perp) = \frac{1}{2\omega_{ip}}[\frac{\not{P}}{M}\omega_{ip} \pm J(i)(m_i + \not{q}_\perp)], \tag{7}$$

$$\varphi_p^{\pm\pm}(q_\perp) \equiv \Lambda_{1p}^{\pm}\frac{\not{P}}{M}\varphi_p\frac{\not{P}}{M}\Lambda_{2p}^{\pm} \tag{8}$$

Constraint conditions: $\varphi^{+-} = \varphi^{-+} = 0$. Instantaneous BS equations:

$$(M - \omega_1 - \omega_2)\varphi^{++} = \Lambda_1^+\eta_p\Lambda_2^+, \quad (M + \omega_1 + \omega_2)\varphi^{--} = -\Lambda_1^-\eta_p\Lambda_2^-. \tag{9}$$

EM decay of X(3872) as a 1^{++} state

The wave function of 1^{++} state can be written as

$$\begin{aligned}\varphi_{1^{++}}(q_\perp) = & i\varepsilon_{\mu\nu\alpha\beta}P^\nu q_\perp^\alpha \varepsilon_1^\beta[\varphi_1 M\gamma^\mu + \varphi_2 \not{P}\gamma^\mu + \varphi_3 \not{q}_\perp\gamma^\mu \\ & + \varphi_4 \not{P}\gamma^\mu \not{q}_\perp/M]/M^2.\end{aligned} \tag{10}$$

For 1^{--} state, the wave function is

$$\begin{aligned}\varphi_{1^-}(q_\perp) = & (q_\perp\cdot\varepsilon)[f_1(q_\perp) + \frac{\not{P}}{M}f_2(q_\perp) + \frac{\not{q}_\perp}{M}f_3(q_\perp) + \frac{\not{P}\not{q}_\perp}{M^2}f_4(q_\perp)] \\ & + M\not{\varepsilon}[f_5(q_\perp) + \frac{\not{P}}{M}f_6(q_\perp) + \frac{\not{q}_\perp}{M}f_7(q_\perp) + \frac{\not{P}\not{q}_\perp}{M^2}f_8(q_\perp)]\end{aligned} \tag{11}$$

We use Cornell potential

$$\begin{aligned}
V(\vec{q}) &= V_s(\vec{q}) + \gamma_0 \otimes \gamma^0 V_v(\vec{q}), \\
V_s(\vec{q}) &= -(\frac{\lambda}{\alpha} + V_0)\delta^3(\vec{q}) + \frac{\lambda}{\pi^2}\frac{1}{(\vec{q}^2+\alpha^2)^2}, \\
V_v(\vec{q}) &= -\frac{2}{3\pi^2}\frac{\alpha_s(\vec{q})}{\vec{q}^2+\alpha^2}, \\
\alpha_s(\vec{q}) &= \frac{12\pi}{25}\frac{1}{\ln(a+\frac{\vec{q}^2}{\Lambda_{QCD}})}.
\end{aligned} \tag{12}$$

where, $a = e = 2.7183$, $\alpha = 0.06$ GeV, $\lambda = 0.2$ GeV, $m_c = 1.7553$ GeV, $m_b =$ 5.13 GeV, $\Lambda_{QCD} = 0.26$ GeV ($c\bar{c}$), 0.20 GeV ($b\bar{b}$). For 1^{++} state, $V_0 =$-0.452 GeV ($c\bar{c}$), -0.521 GeV ($b\bar{b}$), for 1^{--} state, $V_0 =$ -0.465 GeV ($c\bar{c}$), -0.570 GeV ($b\bar{b}$).

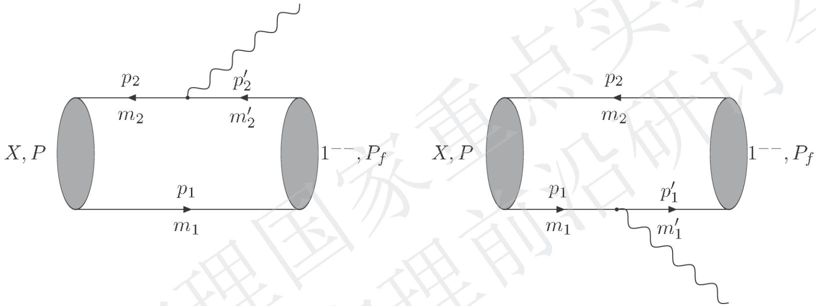

Figure: Feynman diagram of $X(3872)$ EM decay.

Transition amplitude can be written as

$$T = \langle P_f \varepsilon_2, k\varepsilon | S | P\varepsilon_1 \rangle = \frac{(2\pi)^4 e e_q}{\sqrt{2^3 \omega_\gamma E E_f}} \delta^4(P_f + k - P)\varepsilon^\xi M_\xi, \qquad (13)$$

where

$$M^\xi = \frac{e e_q}{(2\pi)^4} \int d^4 q' \int d^4 q \,\mathrm{Tr}\{\bar{\chi}_{P'}(q')\gamma_\mu \chi_P(q)[S_f(q' - \alpha_2{}' P')]^{-1} \delta^4(q - q' + \alpha_2{}' P' - \alpha_2 P) + \bar{\chi}_{P'}(q')[S_f(q + \alpha_1 P)]^{-1}\chi_P(q)\gamma_\mu \delta^4(q - q' + \alpha_1{}' P - \alpha_1' P\prime)\} \qquad (14)$$

$$M^\xi = e e_q \int \frac{d\vec{q}}{(2\pi)^3} \mathrm{Tr}[\frac{\not{P}}{M}\bar{\varphi}'^{++}(q_\perp + \alpha_2 P_{f\perp})\gamma^\xi \varphi^{++}(q_\perp) - \bar{\varphi}'^{++}(q_\perp - \alpha_1 P_{f\perp})\frac{\not{P}}{M}\varphi^{++}(q_\perp)\gamma^\xi], \qquad (15)$$

The positive energy part of wave function for 1^{++} and 1^{--} state can be written as:

$$\varphi_{1^{++}}^{++} = i\varepsilon_{\mu\nu\alpha\beta} P^{\nu} q_{\perp}^{\alpha} \varepsilon_1^{\beta} (A_1 \gamma^{\mu} + A_2 \gamma^0 \gamma^{\mu} + A_3 \gamma^0 \gamma^{\mu} \not{q}_{\perp}), \quad (16)$$

$$\begin{aligned} \varphi_{1^{--}}^{++} = {} & B_1 \not{\varepsilon}_2 + B_2 \not{\varepsilon}_2 \not{P}_f + B_3 \not{P}_f \not{\varepsilon}_2 \not{q}'_{\perp} + B_4 q'_{\perp} \cdot \varepsilon_2 \\ & + B_5 q'_{\perp} \cdot \varepsilon_2 \not{P}_f + B_6 q'_{\perp} \cdot \varepsilon_2 \not{q}'_{\perp} + B_7 q'_{\perp} \cdot \varepsilon_2 \not{P}_f \not{q}'_{\perp}, \end{aligned} \quad (17)$$

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Two-body decay width $\Gamma = \frac{1}{8\pi M}\frac{|\vec{P}_f|}{M}\bar{\Sigma}|T|^2$.

Table: 1^{++} state mass spectrum.

	1P [GeV]	2P	3P
$c\bar{c}$	3.5109	3.9231	4.2220
$b\bar{b}$	9.8929	10.266	10.544

Table: EM decay width of 1^{++} charmonium and bottomonium

	Γ^{1P}_{1S} [keV]	Γ^{2P}_{1S}	Γ^{2P}_{2S}	Γ^{2P}_{3770}
$c\bar{c}$	306	33.0 (33.3)	146 (182)	7.09(9.83)
$b\bar{b}$	30.0	5.65	15.8	

The ratio of different E1 decay widths

$$\frac{\mathrm{Br}(\mathrm{X}(3872)\to\gamma\psi(2\mathrm{S}))}{\mathrm{Br}(\mathrm{X}(3872)\to\gamma J/\psi)}=3.4\pm1.4. \quad (\textit{BaBar}) \tag{18}$$

$$\frac{\mathrm{Br}(\mathrm{X}(3872)\to\gamma\psi(2\mathrm{S}))}{\mathrm{Br}(\mathrm{X}(3872)\to\gamma J/\psi)}=2.1. \quad (\textit{Belle}) \tag{19}$$

From our calculation we can draw

$$\frac{\mathrm{Br}(\mathrm{X}(3872)\to\gamma\psi(2\mathrm{S}))}{\mathrm{Br}(\mathrm{X}(3872)\to\gamma J/\psi)}=4.4, \quad (\textit{prediction}) \tag{20}$$

For $b\bar{b}$

$$\frac{\mathrm{Br}(\chi_{b1}(2\mathrm{P})\to\gamma\Upsilon(2\mathrm{S}))}{\mathrm{Br}(\chi_{b1}(2\mathrm{P})\to\gamma\Upsilon(1\mathrm{S}))}=2.5\pm0.6. \quad (\textit{PDG}) \tag{21}$$

$$\frac{\mathrm{Br}(\chi_{b1}(2\mathrm{P})\to\gamma\Upsilon(2\mathrm{S}))}{\mathrm{Br}(\chi_{b1}(2\mathrm{P})\to\gamma\Upsilon(1\mathrm{S}))}=2.8. \quad (\textit{prediction}) \tag{22}$$

$\Gamma = \frac{1}{8\pi M}\frac{|\vec{P}_f|}{M}\bar{\Sigma}|T|^2$.
Changing potential to get right mass causes little change of wave function. The changing of decay widths comes from both phase space and matrix element.

For $\chi_{c1}(2P) \to \gamma J/\psi$ $|\vec{P}_f|$:
181 MeV to 230 MeV. The whole change: 3.2%. Phase space contributes 0.9%.

For $\chi_{c1}(2P) \to \gamma\varphi(2S)$ $|\vec{P}_f|$:
695 MeV to 736 MeV. The whole change is 23.8%. Phase space contributes 24.7%

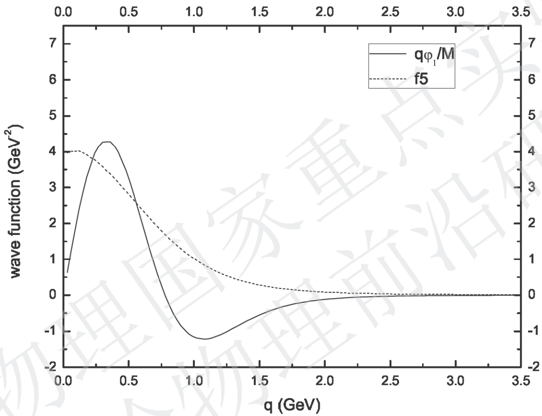

Figure: Radial wave function $\frac{|\vec{q}|}{M}\varphi_1$ of X(3872) and f5 of J/ψ.

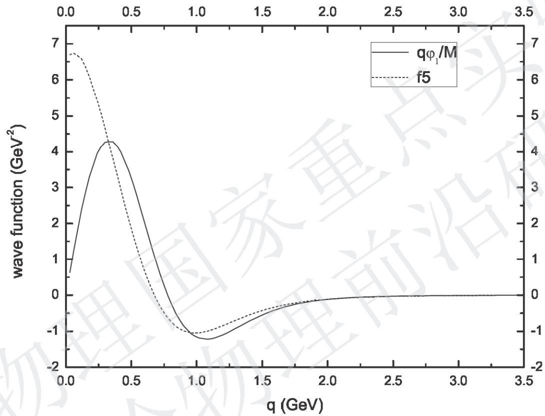

Figure: Radial wave function $\frac{|\vec{q}|}{M}\varphi_1$ of X(3872) and f5 of $\psi(2S)$.

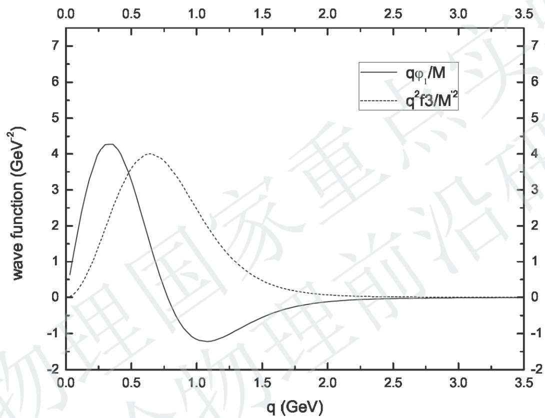

Figure: Radial wave function $\frac{|\vec{q}|}{M}\varphi_1$ of X(3872) and $\frac{|\vec{q}|^2}{M'^2}f3$ of $\psi(3770)$.

EM decay of X(3872) as a 2^{-+} state

Wave function of 2^{-+} state is

$$\varphi_{2^{-+}}(\vec{q}) = \varepsilon_{\mu\nu} q_{\perp}^{\mu} q_{\perp}^{\nu} [f_1(\vec{q}) + \frac{\not{P}}{M} f_2(\vec{q}) + \frac{\not{q}_{\perp}}{M} f_3(\vec{q}) + \frac{\not{P}\ \not{q}_{\perp}}{M^2} f_4(\vec{q})]\gamma_5. \tag{23}$$

The constraint conditions are:

$$\begin{aligned} f_3(q_{\perp}) &= \frac{f_1(q_{\perp}) M (m_1\omega_2 - m_2\omega_1)}{q_{\perp}^2(\omega_1 + \omega_2)}, \\ f_4(q_{\perp}) &= \frac{-M(\omega_1 + \omega_2)}{f_2(q_{\perp})(m_1\omega_2 + \omega_1 m_2)}. \end{aligned} \tag{24}$$

For quark and antiquark have the same mass, f_3 is 0.

Table: 2^{-+} state mass spectrum.

	1D [GeV]	2D	3D	4D
$c\bar{c}$	3.8257	4.1529	4.4063	4.6124
$b\bar{b}$	10.147	10.443	10.681	10.883

$$\begin{aligned}
\Gamma(^1D_2(c\bar{c}) &\to J/\psi\gamma) = 1.22\times 10^{-6}GeV, \\
\Gamma(^1D_2(c\bar{c}) &\to \psi(2S)\gamma) = 1.69\times 10^{-10}GeV, \\
\Gamma(^1D_2(c\bar{c}) &\to \psi(3770)\gamma) = 3.41\times 10^{-13}GeV, \\
\Gamma(^1D_2(c\bar{c}) &\to h_c(1P)\gamma) = 3.45\times 10^{-4}GeV.
\end{aligned} \tag{25}$$

Conclusions

- X(3872) as a $\chi_{c1}(2P)$ state can explain EM decay quite well. But there still needs more data to make the final judgement.
- It's not likely to be a $^1D_2$ (2^{-+}) state.

Wave function construction

Under parity and charge conjugate spinor wave function transforms as

$$
\begin{aligned}
\hat{C}\psi(x)\hat{C}^{-1} &= \eta_c^s C\bar{\psi}^T \\
\hat{P}\psi(x)\hat{P}^{-1} &= \eta_p^s P\psi \\
\hat{C}|\vec{P},s> &= \eta_c|\vec{P},\bar{s}> \\
\hat{P}|\vec{P},s> &= \eta_p|-\vec{P},E,s>
\end{aligned}
\tag{26}
$$

We define BS wave function

$$
\chi_{P\xi}(x_1,x_2) = <0|T[\psi(x_1)\bar{\psi}(x_2)]|P\xi> \tag{27}
$$

Under parity and charge conjugate we have,

$$
\begin{aligned}
\chi_{\vec{P},s}(x) &= \eta_c C\chi^T_{\vec{P},s}(-x)C^{-1}, \\
\chi_{\vec{P},s}(x) &= \eta_p\gamma_0\chi_{-\vec{P},E,s}(-\vec{x},t)\gamma_0
\end{aligned}
\tag{28}
$$