Light Composite Higgs Made of Heavy Vector-like Fermions

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1. Spontaneous Breaking of Vector Symmetries and the Nondecoupling Light Higgs Particle
   Phys. Rev. D52 (1995)6500
2. Low Energy Properties of the Heavy Vector Fermions
   Phys. Rev. D51(1995)251
3. A Renormalization Group Analysis of the Higgs Boson with Heavy Fermions and Compositeness
   Phys. Lett. B378(1996)201; Erratum - ibid. B382(1996)448

Situation in year 1995 $\cdots$

1. Physics at Electro-weak scale is well described by an effective theory called the Standard Model
2. The effective theory is written down in terms of a renormalizable lagrangian model.
3. Higgs was missing, and no fundamental scalar had yet been observed in Nature.

No fundamental scalar$\longrightarrow$no Higgs$\longrightarrow$ no Standard model but standard model is correct$\longrightarrow$ so there is a Higgs and most probably be light $\longrightarrow$ there is a fundamental scalar contradicting basic observation on Nature$\longrightarrow$ No fundamental scalar$\cdots\cdots$

Spontaneous breaking of vector-like symmetry

Using a four fermion interaction Lagrangian, we demonstrate that the spontaneous breaking of vector symmetries requires the existence of a light (comparing with the heavy fermion mass) scalar particle and the low energy effective theory (the σ model) obtained after integrating out heavy fermion degrees of freedom is asymptotically a renormalizable one. When applying the idea to the electroweak symmetry breaking sector of the standard model, the Higgs particle's mass is of the order of the electroweak scale.

A four-fermi interaction version of the “Parity Doublet” model

$$\mathcal{L} = \bar{\Psi}^i \left(i\partial\!\!\!/ - M\right) \Psi^i - \frac{G}{N_c \Lambda^2}[(\bar{\Psi}^i \rho_3 \Psi^i)^2 + (\bar{\Psi}^i \rho_1 \vec{\tau} \Psi^i)^2] \ , \qquad (1)$$

where $\Psi = (\psi_1, \psi_2)^T$ and $\psi_{1,2}$ are SU(2) ‘isospin’ doublets.

Invariant under the following $SU(2) \times SU(2)$ rotations:

$$\Psi \rightarrow e^{i\vec{\alpha}\cdot\vec{\tau} + i\rho_2 \vec{\beta}\cdot\vec{\tau}} \Psi \ . \qquad (2)$$

To match the electroweak physics one of the $SU(2)$ global symmetry will be gauged as $SU(2)_W$ (local $U(1)_Y$ should also be introduced). The another "custodial" $SU(2)$ symmetry remains as a global one and can be broken explicitly but slightly.

Gap Equation and SBVS

$m = m_s + \rho_3 m_3$.

$$m_s = M \ , \tag{3}$$

$$m_3 = \frac{iG}{\Lambda^2} \int^{\Lambda} \frac{d^4 p}{(2\pi)^4} tr(\rho_3 S_F) \ , \tag{4}$$

The above equation (4) can be written in a simple form. To define

$$f(m) = \frac{m}{\Lambda^2} \int^{\Lambda} \frac{q_E^2 dq_E^2}{q_E^2 + m^2} \ , \tag{5}$$

we have,

$$m_1 - m_2 = \frac{G}{\pi^2} [f(m_1) - f(m_2)] \ , \tag{6}$$

where $m_{1,2} = M \pm m_3$.

For small but non-vanishing $m_1 - m_2$:

$$\frac{\pi^2}{G} \simeq f'(M) + 1/6 m_3^2 f'''(M) \ . \tag{7}$$

when π^2/G is smaller than unity there exists the critical value M_c, $\pi^2/G = f'(M_c)$. When M is less than M_c there exists a non-vanishing solution of m_3,

$$m_3 = \sqrt{6M_c(M_c - M)} \ , \tag{8}$$

which holds in the $m_3 << M$ (or $M \to M_c$) limit. Once M exceeds the critical value M_c there is only the trivial solution $m_3 = 0$ in the above equation (7).

Less fine tuning!

J. P. Preskill and S. Weinberg, Phys. Rev. **D24** (1981) 1059.

Persistent mass condition

To understand more about the dynamics of SBVS it is necessary to solve the Lagrangian eq. (1) in the large N_c limit:

$$\Pi_P(q^2) \equiv i \int d^4x e^{iqx} < | T\{\bar{\Psi}^i(x)\rho_1\Psi^j(x)\bar{\Psi}^j(0)\rho_1\Psi^i(0)\}| > ,$$

$$\Pi_S(q^2) \equiv i \int d^4x e^{iqx} < | T\{\bar{\Psi}(x)\rho_3\Psi(x)\bar{\Psi}(0)\rho_3\Psi(0)\}| > ,$$

$$\Pi^{\mu}_M(q^2) \equiv i \int d^4x e^{iqx} < | T\{\bar{\Psi}^i(x)\rho_2\gamma^{\mu}\Psi^j(x)\bar{\Psi}^j(0)\rho_1\Psi^i(0)\}| >$$

$$\equiv iq^{\mu}\Pi_M(q^2) .$$

$$\Pi_P(q^2) = \frac{\overline{\Pi}_P(q^2)}{1 - G/\Lambda^2 \overline{\Pi}_P(q^2)} \ , \tag{9}$$

$$\Pi_S(q^2) = \frac{\overline{\Pi}_S(q^2)}{1 - G/\Lambda^2 \overline{\Pi}_S(q^2)} \ , \tag{10}$$

$$\Pi^\mu_M(q) = \frac{\overline{\Pi}^\mu_M(q)}{1 - G/\Lambda^2 \overline{\Pi}_P(q^2)} \ , \tag{11}$$

$\Rightarrow$

$$\Pi^\mu_M \equiv \frac{2iq^\mu}{q^2} < |\overline{\Psi}\rho_3\Psi| > \ . \tag{12}$$

$$< |\overline{\Psi}\rho_3\Psi| > = -\frac{N_c}{2G}\Lambda^2(m_1 - m_2) \ . \tag{13}$$

Higgs Mass

$$m_H \simeq 2m_3 .$$

Two remarks:

1. Higgs mass is not roughly twice of its constituents' mass.
2. Bosonization via Heat Kernel Expansion technique. LEFT is asymptotically renormalizable with a LIGHT Higgs (mass naturally around electroweak scale.)

$v^2 = f_\pi^2 = \frac{N_c}{2\pi^2} m_3^2 \ln(\Lambda^2/M^2)$

$$m_H = \frac{2\pi v}{\sqrt{N_c \ln(\Lambda/M)}} . \tag{14}$$

Taking for example $\Lambda/M \simeq 10$ we may obtain the upper bound of the Higgs particle's mass and taking $\Lambda \sim 10^{18}$GeV and $M \sim 10^3$GeV the lower bound may be estimated, we have,

$$185/\sqrt{N_c}GeV \leq m_H \leq 720/\sqrt{N_c}GeV \ . \tag{15}$$

RGE analysis (I)

It is helpful, not to integrate out fermion fields completely but firstly down to an arbitrary scale μ to study the heavy fermion contributions to the running coupling constants of the composite Higgs field. We have,

$$\lambda_0(\mu) = \frac{N_c}{8\pi^2}\ln(\frac{\Lambda^2+M^2}{\mu^2+M^2}) \ , \quad Z_H(\mu) = \frac{N_c}{4\pi^2}\ln(\frac{\Lambda^2+M^2}{\mu^2+M^2}) \ , \qquad (16)$$

$$m_H^2(\mu) = \frac{N_c}{2\pi^2}\{\frac{\pi^2}{G}\Lambda^2 - \Lambda^2 + \mu^2 + 3M^2\ln(\frac{\Lambda^2+M^2}{\mu^2+M^2})\} \ . \qquad (17)$$

Only the high frequency modes ($\mu > M$) contribute to the wave function renormalization constant (Z_H) and the bare coupling constant of ϕ^4 self interactions (λ_0). The low frequency modes only contribute to the fine tuning of the Higgs mass.

Equivalence to the Standard Model

Once introducing the matter field (quarks and leptons) couplings in the same way as in the SM we can set up the complete equivalence between the SM and our model of SBVS, equation (1) in the $m/M << 1$ limit, even at the energy scale E much larger than the electroweak scale as long as $E << M$, within the constraints on the Higgs particle's mass.

RGE analysis (II)

$$\mathcal{L} = \bar{Q}(i\not{D}_d - M)Q + \bar{U}(i\not{D}_s - M)U + \bar{D}(i\not{D}_s - M)D$$
$$+\{g_Y \bar{Q}\phi D + g'_Y \bar{Q}\tilde{\phi} U + h.c.\} \ . \qquad (18)$$

In above Lagrangian we introduced four vector-like fermions, Q is a $SU(2)_W$ doublet and U and D are singlets. We assume they participate in strong interactions and are in fundamental representations of $SU(3)_C$. They are equivalent to one family of chiral quarks plus a left–right conjugated chiral quark family.

The correction to the effective potential:

$$\delta V(\phi_c) = -\frac{N_c}{16\pi^2}\{(M+g_Y\phi_c)^4\log(\frac{(M+g_Y\phi_c)^2}{\mu^2}) + (M-g_Y\phi_c)^4\log(\frac{(M-g_Y\phi_c)^2}{\mu^2})\} + (g_Y \to g'_Y) .$$

Because of the negative sign, fermions turn to destabilize the vacuum. At a scale $\phi_c < M$ one can expand the above expression in powers of ϕ_c^2/M^2 and it is easy to verify that heavy fermions decouple from the effective potential as a consequence of the decoupling theorem. Far above the threshold there is no difference between chiral and vector-like fermions. The only essential ingredient is the number of independent Yukawa couplings.

Compositeness Condition

The effective Yukawa interaction Lagrangian is identical to the standard model at the cutoff scale Λ, but with vanishing wave function renormalization constant of the Higgs field ($Z_H = 0$) and vanishing Higgs self-coupling ($\lambda = 0$). Below Λ the model is equivalent to the standard model and therefore the coupling constants of the effective theory run according to the standard model renormalization group equations. However the vanishing of Z_H at the scale $\mu = \Lambda$ leads to the following boundary conditions of the renormalization group equations:

$$g_Y^r \to \infty \, , \quad \lambda^r/(g_Y^r)^4 \to 0 \tag{19}$$

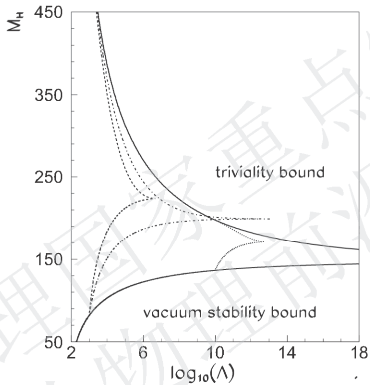

Figure: Adding vector-like fermions can rescue top condensate model

结束语 I

All puzzles raised in the first two slides seemed to be answered

1. Why the electro-weak physics is described by a renormalizable theory?
2. Why a light Higgs?
3. Why it is pointless to question an ‘elementary’ scalar filed?

展望未来..........

结束语 II

“心即理”

—— 王守仁（1472-1529）

心？— Standard Model （？）

理？— Planck Scale Random Dynamics（？）

心即理：— A kind of duality

不仅 building block of nature, 就连 symmetry 都是从 random dynamics 里产生出来的。只有这样，才会有一个真正的终极理论！

Thanks for patience!