Charmless B meson decays in the pQCD approach

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Outlines

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- ♠ Works at Leading Order
- ♠ NLO contributions in pQCD
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1. Introduction

With the great progress in both theory and experiments during the past thirty years, nowadays, B physics becomes one of the most active research areas in high energy physics.

Motivation

- The experimental measurements and theoretical studies of $B \to M_1 M_2$ decays play an important role in the precision test of the SM and in searching for NP beyond the SM.

  — T. Mannel, A. Soni's talk; R. Fleischer's talk; C.S. Kim's talk

- The BaBar and Belle collaborations have been collected more than

1200 M events of $B\bar{B}$ pair prod. and decays. Many decays, such as $B \to \pi\pi, K\pi, \rho\pi, \cdots$ have been measured with good precision. Much more B meson events ($B_{u,d}, B_s, B_C$ and $b-$hadrons) are expected at the forthcoming LHC experiments.

- In the SM, the decay amplitude can be written as

$$\mathcal{A}(B \to M_1 M_2) = \frac{G_F}{\sqrt{2}} \sum_i \lambda_i \, C_i(\mu) < M_1 M_2 | O_i(\mu) | B > . \qquad (1)$$

The Wilson coefficients are known at NLO level;

TH uncertainties from the evaluation of $< M_1 M_2 | O_i(\mu) | B >$ are , unfortunately , still rather large. One has to pin down the TH.errors of SM predictions, before one can find/comfirm the new physics signals.

Factorization Approaches

In recent years, great progress has been made in evaluating hadronic matrix elements, and the popular factorization approaches include:

- QCD Factorization [B B N S, D.S.Du et al.,];

- pQCD Factorization [H.N. Li, C.D. LÜ, A.I. Sanda, Y.Y. Keum, M.Z. Yang, et al.,];

  — Hsiang-nan's forthcoming talk

- The soft-collinear effective theory [Bauer, Pirjol, Rothstein, Stewart, Beneke, Yang, Jager, et al.,].

  — D.S. Yang's talk

2. Outline of the pQCD Approach

♣ Based on Lepage and Brodsky's work (PR D22(1980)2157), Hsiang-nan and several other researchers developed the so-called pQCD factorization approach to calculate the hadronic matrix elements of the $B \to M_1 M_2$ decays.

♣ In pQCD approach, since B meson is heavy, the final two light mesons have large energy. One assumes that such process dominated by the exchanges of hard gluons, and then the hard part $H(t, x_i)$ can be separated and calculated perturbatively. The soft part can be absorbed into wave-functions $\Phi_{M_i}(x_i)$ of B and final state mesons.

♣ The FSI should be small, because of Bjorken's Color-transparency mechanism.

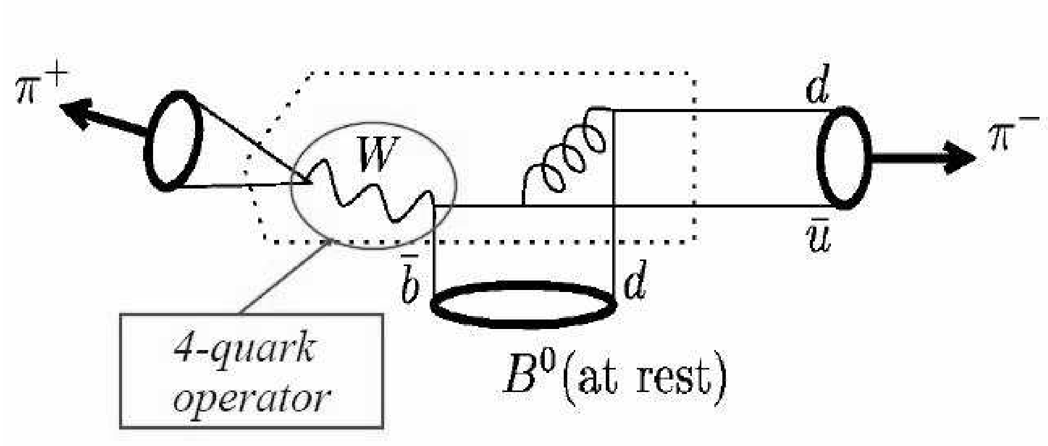


Figure 1: $B^0 \to \pi^+\pi^-$ decay. 4-q operators inside the circle.

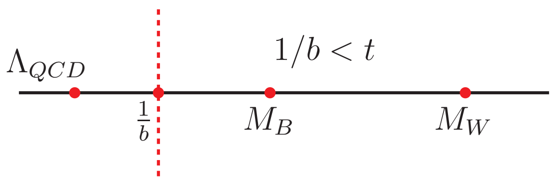


Figure 2: The typical energy scales relevant in B decay process. $m_w \gg m_b > \sqrt{\Lambda m_B} > 1/b_i > \Lambda_{QCD}$,

♣ In pQCD, $\mathcal{A}(B \to M_1 M_2)$ can be conceptually written as the convolution:

$$\mathcal{A} \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \, \mathrm{Tr}\left[C(t)\Phi_B(x_1, b_1) \Phi_{M_1}(x_2, b_2)\Phi_{M_2}(x_3, b_3) H(x_i, b_i, t) \cdot S_t(x_i) \cdot e^{-S(t)}\right], \quad (2)$$

— $C(t)$, Wilson coefficients, short-distance contribution, RG running;

— $H(x_i, b_i, t)$, hard kernel, perturbatively calculated;

— $e^{-S(t)}$, Sudakov suppression factor, which suppresses the soft dynamics effectively;

— $S_t(x_i)$, suppression factor, obtained from threshold resummation, which smears the end-point singularities on x_i;

♣ In pQCD, the k_T in the denominator is kept, which can kill the end-point singularity effectively. [Li, PPNP 51, 85 (2003) and reference therein.]

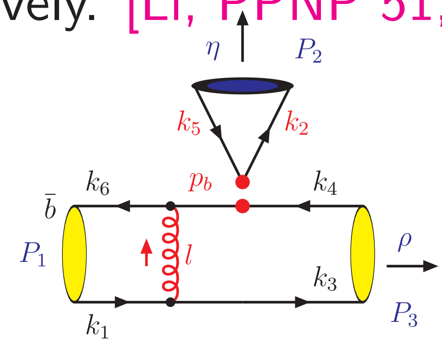


Figure 3: $B \longrightarrow \rho$ form factor

♣ There are two propagators, gluon and quark propagators respectively. If neglecting $\mathbf{k}_T$, we will find

$$\frac{1}{l^2} \cdot \frac{1}{p_b^2 - M_B^2} = \frac{1}{(k_1 - k_3)^2} \frac{1}{(P_1 - k_3)^2} = -\frac{1}{M_B^4 x_1 x_3 (1 - x_3)}. \tag{3}$$

The end-point singularity will appear for $x_1 \to 0$ or $x_3 \to 0, 1$;

If we keep $\mathbf{k}_T$ in the end-point region, we find

$$\frac{1}{M_B^4 x_1 x_3 (1-x_3)} \rightarrow \frac{1}{((1-x_3)M_B^2 + \mathbf{k}_{3T}^2)(x_1 x_3 M_B^2 + (\mathbf{k}_{1T} - \mathbf{k}_{3T})^2)} \rightarrow \frac{1}{\mathbf{k}_{3T}^2 \cdot |\mathbf{k}_{1T} - \mathbf{k}_{3T}|^2} \quad (4)$$

The end-point singularity will disappear!

♣ In pQCD, the form factors can be calculated perturbatively, the only inputs are the wave functions of the mesons involved.

LO Feynman Diagrams

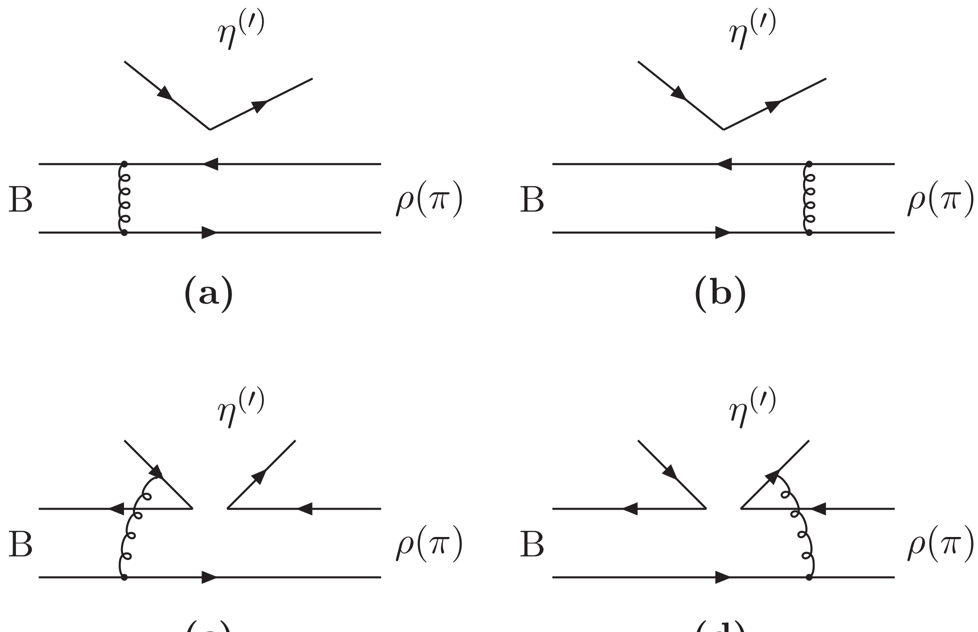


Figure 4: Emission diagrams for $B \to \rho(\pi)\eta^{(\prime)}$ decays. The form factors $F_{0,1}^{B\to\rho}(0)$ and $F_{0,1}^{B\to\pi}(0)$ can be extracted from the factorizable emission diagram (a) and (b).

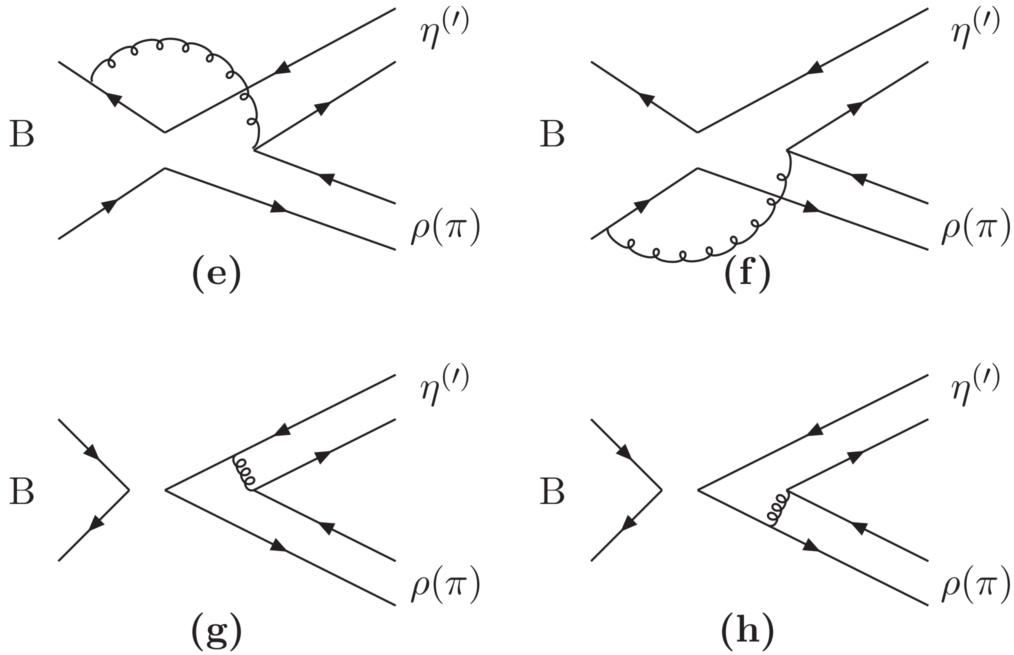


Figure 5: The annihilation diagrams for $B \to \rho(\pi)\eta^{(\prime)}$ decays. Which can contribute a large strong phase.

♣ For dynamics, see Li's review papers and Lectures:

- Early papers: "Three-scale factorization theorem and ⋯" , C.H.V. Chang, H.-n Li, Phys. Rev. D 55 (1997)5577;
  "Factorization theorem, EFT, NL heavy meson decays", T.W. Yeh, H.-n Li, Phys. Rev. D 56 (1997)1615.

- Hsiang-nan's review paper:
  QCD Aspects of Exclusive B meson decays, PPNP, 51 (2003) 85-171. Detailed discussion about the pQCD dynamics and phenomenology.

- Series of Lectures given by Hsiang-nan Li in Beijing, in summer of 2005; and in Nanjing, Dec. 2006;

- Recent talks given by C.D. Lü, at Nanjing, Taiwan, Italy, etc.

3. Works at Leading Order

Since 2001, almost all $B/B_s \to PP, PV, VV$ decays have been calculated in the pQCD approach at leading order:

Research Groups and Members

- Hsiang-nan Li, A.I. Sanda, S. Mishima, K. Ukai, Y.Y. Keum , T. Kurimoto, A. Ali, G. Kramer, Y.Y.Charng, C.H. Chen, et al;
- C.D. Lü, Z.J. Xiao, M.Z. Yang, Libo Guo, Y. Li, X.Q Yu, Y.L.Shen, Y.M. Wang, Wei Wang, Xin Liu, et al;
- Around 100 papers have been published since 2001!

$B/B_s \to M_1 M_2$, pQCD predictions:

♣ For branching ratios, the pQCD predictions are consistent with those obtained based on the QCDF, or SCET approach.

Phys. Lett. B 504(2001)6; Phys. Rev. D 63, 054008(2001);
Phys. Rev. D 63, 074009(2001); Phys. Rev. D 64, 112002 (2001);
Eur. Phys. J. C 24(2002)121; Phys. Rev. D 66, 054013(2002);
Eur. Phys. J. C 41(2005)311; Phys. Rev. D 72, 054015(2005);
Phys. Rev. D 70, 034009(2004).

♣ For $B_s \to M_1 M_2$ decays, the LO pQCD calculations of around fifty decay modes have been done.

A. Ali, G. Gramer, C.D Lü, et al., Phys. Rev. D 76, 014008 (2007) ;
Z.J. Xiao et al., Phys. Rev. D 75, 034017 (2007); EPJC 50 (2007)363;

$K\eta'$ puzzle, LO pQCD predictions.

Decays	HFAG(10^{-6})	pQCD-2002	Decays	HFAG(10^{-6})
$K^+\eta$	2.7 ± 0.3	$-$	$K^{*+}\eta$	15.9 ± 1.0
$K^+\eta'$	70.2 ± 2.5	$-$	$K^{*+}\eta'$	3.8 ± 1.2
$K^0\eta$	< 1.9	~ 4.6	$K^{*0}\eta$	19.3 ± 1.6
$K^0\eta'$	64.9 ± 3.2	~ 45	$K^{*0}\eta'$	4.9 ± 2.1

♣ For $B \to K\eta^{(\prime)}, K^*\eta^{(\prime)}$ decays, it is very difficult to interpret the observed pattern of Br's [Kou, Sanda, PLB 525(2002)240 in pQCD]:

♣ The contribution induced by $q\bar{q}$ $(q = u, d)$ component and $s\bar{s}$ one interferes destructively for $B \to K\eta$ decay, but constructively for $B \to K\eta'$ decays. For $B \to K^*\eta^{(\prime)}$ decays, the interference is reflected!

$B \to M_i \eta^{(\prime)}$ ($M_i \neq K$), Agree well!

Table 1: pQCD predictions for BR's (10^{-6}). Xin Liu et al., Phys. Rev. D 73, 074002(2006); H.S. Wang et al., Nucl. Phys. B 738 (2006) 243;

Decays	pQCD	QCDF	HFAG
$B^{\pm} \to \rho^{\pm}\eta$	$8.5^{+3.4}_{-2.3}$	$9.4^{+5.9}_{-4.8}$	5.4 ± 1.2
$B^{\pm} \to \rho^{\pm}\eta'$	$8.7^{+3.3}_{-2.7}$	$6.3^{+4.0}_{-3.3}$	$9.1^{+3.7}_{-2.8}$
$B^0 \to \rho^0\eta$	$0.02^{+0.10}_{-0.02}$	$0.03^{+0.17}_{-0.10}$	< 1.5
$B^0 \to \rho^0\eta'$	$0.06^{+0.12}_{-0.02}$	$0.01^{+0.12}_{-0.06}$	< 1.3
$B^{\pm} \to \pi^{\pm}\eta$	$4.1^{+1.5}_{-1.1}$	$4.7^{+2.7}_{-2.3}$	4.4 ± 0.4
$B^{\pm} \to \pi^{\pm}\eta'$	$2.4^{+0.9}_{-0.6}$	$3.1^{+1.9}_{-1.7}$	$2.7^{+0.6}_{-0.5}$
$B^0 \to \pi^0\eta$	0.23 ± 0.08	$0.28^{+0.48}_{-0.28}$	< 1.3
$B^0 \to \pi^0\eta'$	0.19 ± 0.05	$0.17^{+0.33}_{-0.17}$	1.5 ± 0.7

Table 2: The pQCD predictions for BR's. Z.J. Xiao et al., Phys. Rev. D 75, 014018 (2007); Phys. Rev. D 75, 054033 (2007).

Decays	pQCD	QCDF	HFAG
$B^0 \to \omega\eta$	$\left(2.7^{+1.1}_{-1.0}\right) \times 10^{-7}$	$3.1^{+4.6}_{-2.7} \times 10^{-7}$	$< 17 \times 10^{-7}$
$B^0 \to \omega\eta'$	$\left(0.75^{+0.37}_{-0.33}\right) \times 10^{-7}$	$\left(2.0^{+3.4}_{-1.8}\right) \times 10^{-7}$	$< 28 \times 10^{-7}$
$B^0 \to \phi\eta$	$\left(6.3^{+3.3}_{-1.9}\right) \times 10^{-9}$	1×10^{-9}	$< 6 \times 10^{-7}$
$B^0 \to \phi\eta'$	$\left(7.5^{+3.5}_{-2.6}\right) \times 10^{-9}$	1×10^{-9}	$< 10 \times 10^{-7}$
$B^0 \to \eta\eta$	$\left(0.67^{+0.32}_{-0.25}\right) \times 10^{-7}$	$\left(1.6^{+4.5}_{-1.9}\right) \times 10^{-7}$	$< 1.8 \times 10^{-6}$
$B^0 \to \eta\eta'$	$(0.18 \pm 0.11) \times 10^{-7}$	$\left(1.6^{+6.1}_{-1.8}\right) \times 10^{-7}$	$< 1.7 \times 10^{-6}$
$B^0 \to \eta'\eta'$	$\left(0.11^{+0.12}_{-0.09}\right) \times 10^{-7}$	$\left(0.6^{+2.5}_{-0.7}\right) \times 10^{-7}$	$< 2.4 \times 10^{-6}$

Extraction of Form factors

♣ The form factors can be calculated perturbatively in pQCD approach, extracted from the first two factorizable emission diagrams, and are consistent in value with those obtained from the QCD sum-rule:

— For $B \to P$, $B \to V$ with zero-momentum transfer:

$$\begin{aligned} F_{0,1}^{B\to\pi} &= 0.24; \quad F_{0,1}^{B\to K} = 0.36; \quad F_{0,1}^{B\to\eta^{(\prime)}} = 0.22; \\ A_0^{B\to K^*} &= 0.38; \quad A_1^{B\to K^*} = 0.24; \quad V^{B\to K^*} = 0.31; \\ A_0^{B\to\rho} &= 0.32; \quad A_1^{B\to\rho} = 0.21; \quad V^{B\to\rho} = 0.27; \\ A_0^{B\to\omega} &= 0.29; \quad A_1^{B\to\omega} = 0.17; \quad V^{B\to\omega} = 0.24; \end{aligned} \tag{5}$$

— For $B_S \to P, V$ transition:

$$\begin{aligned} F_{0,1}^{B_s\to K} &= 0.25; \quad F_{0,1}^{B_s\to \eta} = 0.37; \\ A_0^{B_s\to K^*} &= 0.29; \quad A_1^{B_s\to K^*} = 0.18; \quad V^{B_s\to K^*} = 0.24; \\ A_0^{B_s\to \phi} &= 0.34; \quad A_1^{B_s\to \phi} = 0.21; \quad V^{B_s\to \phi} = 0.28; \end{aligned} \tag{6}$$

♣ The theoretical uncertainties are about 20%, mainly from parameter ω_b, and from the wave functions of the light final state mesons.

♣ In calculations, the possible gluonic contributions are ignored, since they are generally small?

Y.Y. Charng, T. Kurimoto, and H.N. Li, *"Gluonic contribution to $B \to \eta^{(\prime)}$ form factors"*, Phys. Rev. D **74**, 074024 (2006).

CP-violating Asymmetries

♣ BR's, CPV Asymmetries, $B \to K\pi$ puzzle
Keum, Li, and Sanda, Phys.Rev. D63, 054008 (2001);
C.D. Lü, K. Ukai and M.Z. Yang, Phys.Rev. D63, 074009 (2001);
H.-n Li, S. Mishima, A.I. Sanda, Phys. Rev. D **72**, 114005(2005).

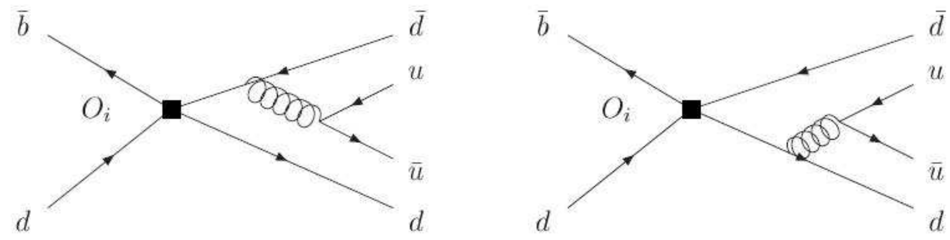
Table 3: CP-violating asymmetries(10^{-2}).

Decays	QCDF	pQCD-LO	pQCD-NLO	Data
$K^+\pi^-$	$+5 \pm 9$	-12	-9^{+6}_{-8}	-9.7 ± 1.2
$K^+\pi^0$	$+7 \pm 9$	-8	-1^{+3}_{-5}	4.7 ± 2.6
$K^0\pi^+$	1 ± 1	-1	0 ± 0	0.9 ± 2.5
$\pi^+\pi^-$	-6 ± 12	$+14$	18^{+20}_{-12}	$+38 \pm 7$

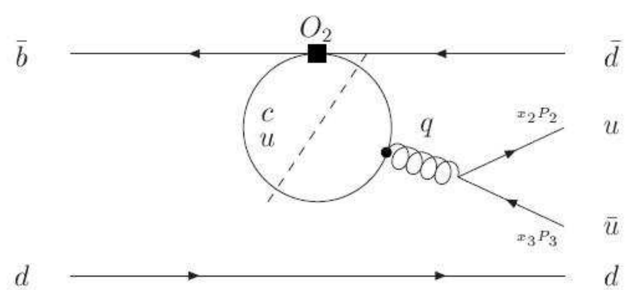
♣ The pQCD predictions for CPV, are generally large! The mechanism to produce CPV are different in pQCD and QCDF/SCET!

In QCDF, BSS mechanism(Bander, Silverman, Soni: PRL 43(1979)242): the strong phase comes from the imaginary part of the internal on-shell quark loop in the gluon propagator!

One should also remember that the Th. predictions for CPV still have large Th. errors. FSI may also be important for CPV.



(a)



(b)

Figure 6: (a) pQCD, annihilation; (b) BSS mechanism.

4. Next-to-Leading Order contributions in pQCD

♣ The NLO contributions to H, including the vertex corrections, the quark loops, and the chromo-magnetic penguin.

♣ NLO contributions to $B \to K\pi$ and some $B \to PV$ decays;
[Li, Mishima, Sanda, PR D72, 114005(2005), PR D74, 094020 (2006)].
[Dandi, Li, "NLO corrections to excl. process in k_T fac.", PR D76, 034008 (2007)].

♣ My group is now calculating the NLO contributions to some $B/B_s \to M_1M_2$ decays in the pQCD approach.

$$B \to (K, K^*)\eta^{(\prime)}; \quad KK, KK^*, (\rho, \omega, \phi, \pi)\eta^{(\prime)};$$
$$B_s \to PP, PV; \tag{7}$$

NLO Wilson coefficients $C_i(\mu)$

♣ We now use the NLO Wilson coefficients $C_i(\mu)$ and the RG evolution at the NLO level.

$$U(m_1, m_2, \alpha) = U(m_1, m_2) + \frac{\alpha}{4\pi} R(m_1, m_2) \quad (8)$$

where the function $U(m_1, m_2)$ and $R(m_1, m_2)$ represent the QCD and QED evolution [Buchalla, Buras, Lautenbacher, Rev.Mod.Phys.68 (1996) 1125]. We also set a cut-off scale $\mu_0 = 1$ GeV, instead of $\mu_0 = 0.5$ GeV as being used by Li.

Table 4: The numerical values of the LO and NLO Wilson coefficients $C_i(m_b)$, $C_{7\gamma}(m_b)$ and $C_{8g}(m_b)$.

$C_i(m_b)$	C_1	C_2	C_3	C_4	C_5	C_6
LO	-0.2812	1.1246	0.0130	-0.0278	0.0080	-0.0343
NLO	-0.1747	1.0774	0.0125	-0.0330	0.0094	-0.0393
$C_i(m_b)$	C_7/α	C_8/α	C_9/α	C_{10}/α	$C_{7\gamma}$	C_{8g}
LO	0.1338	0.0514	-1.1459	0.2865	-0.3109	-0.1481
NLO	-0.0032	0.0305	-1.2760	0.2553	-0.3016	$--$

♣ In the region $m_b \le t \le m_W$, $N_f = 5$, take $C_i(m_W)^{NLO}$ as input;

In the region $\mu_0 \le t < m_b$, $N_f = 4$, take $C_i(m_b)^{NLO}$ as input;

In the region $t < \mu_0$, we fix the values $C_i(t)$ at $C_i(\mu_0)$.

♣ We also use the formula at two-loop level for the running of $\alpha_s(t)$

$$\alpha_s(t) = \frac{4\pi}{\beta_0 \ln\left[t^2/\Lambda^2_{QCD}\right]} \cdot \left\{ 1 - \frac{\beta_1}{\beta_0^2} \cdot \frac{\ln\left[\ln\left[t^2/\Lambda^2_{QCD}\right]\right]}{\ln\left[t^2/\Lambda^2_{QCD}\right]} \right\}, \tag{9}$$

where $\beta_0 = (33 - 2N_f)/3$, $\beta_1 = (306 - 38N_f)/3$, $\Lambda^{(5)}_{QCD} = 0.225$ GeV and $\Lambda^{(4)}_{QCD} = 0.326$ GeV.

Vertex corrections

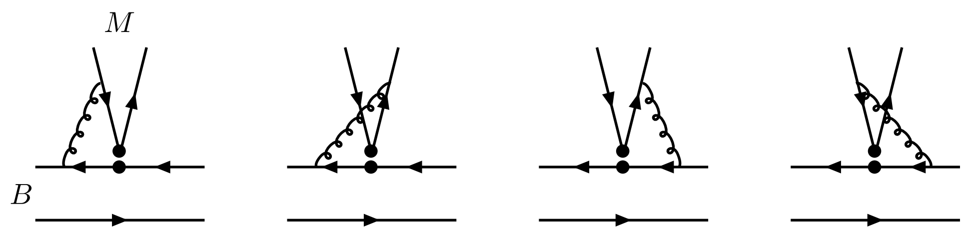


Figure 7: NLO vertex corrections to factorizable emission diagrams.

♣ V-corrections to above emission diagrams have been calculated years ago in the QCDF [BBNS, PRL 1999, Nucl. Phys. B 675(2003)333].

♣ The difference induced by setting $k_t \neq 0$ is less than 10%, and can be neglected. We use the relevant formulas as given in QCDF directly.

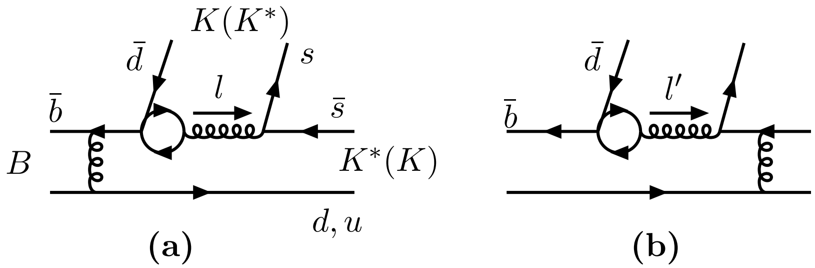
♣ The vertex corrections can be absorbed by adding a vertex-function $V_i(M)$ to a_i

$$\begin{aligned} a_i(\mu) &\rightarrow a_i(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_F \frac{C_i(\mu)}{3} V_i(M), \quad for \quad i = 1, 2; \\ a_j(\mu) &\rightarrow a_j(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_F \frac{C_{j\pm 1}(\mu)}{N_c} V_j(M), \quad for \quad j = 3 - 10, \end{aligned} \quad (10)$$

where M is the meson emitted. For a pseudo-scalar M,

$$V_i(M) = \begin{cases} 12 \ln \frac{m_b}{\mu} - 18 + \frac{2\sqrt{2N_c}}{f_M} \int_0^1 dx \phi_M^A(x) g(x), & \text{for} \quad i = 1-4, 9, 10, \\ -12 \ln \frac{m_b}{\mu} + 6 - \frac{2\sqrt{2N_c}}{f_M} \int_0^1 dx \phi_M^A(x) g(\bar{x}), & \text{for} \quad i = 5, 7, \\ -6 + \frac{2\sqrt{2N_c}}{f_M} \int_0^1 dx \phi_M^P(x) h(x), & \text{for} \quad i = 6, 8, \end{cases} \quad (11)$$

Quark-loop contributions



♣ The contributions from quark loops are described by $H_{eff}^{(q)}$

$$H_{eff}^{(q)} = - \sum_{q=u,c,t} \sum_{q\prime} \frac{G_F}{\sqrt{2}} V_{qb} V_{qd}^* \frac{\alpha_s(\mu)}{2\pi} C^{(q)}(\mu, l^2) \cdot \left(\overline{d}\gamma_\rho (1-\gamma_5) T^a b\right) \cdot \left(\overline{q}'\gamma^\rho T^a q'\right), \quad (12)$$

♣ Take $B \to KK^*$ as an example: for the case of $B \to K^*$ transition, the decay amplitude $M^{(q)}_{K^*K}$ with $q = u, c, t$ are

$$
\begin{aligned}
M^{(q)}_{K^*K} \quad = \quad & -\frac{4}{\sqrt{3}} G_F C_F^2 m_B^4 \int_0^1 dx_1 dx_2\, dx_3 \int_0^\infty b_1 db_1 b_2 db_2\, \phi_B(x_1, b_1) \\
& \cdot \Big\{ \big\{ (1 + x_2)\, \phi_{K^*}(\bar{x}_2) \phi_K^A(\bar{x}_3) \\
& - r_{K^*} (1 - 2x_2) \left[\phi^s_{K^*}(\bar{x}_2) - \phi^t_{K^*}(\bar{x}_2) \right] \phi_K^A(\bar{x}_3) \\
& - 2 r_K \phi_{K^*}(\bar{x}_2) \phi_K^P(\bar{x}_3) \\
& + 2 r_{K^*} r_K \left[(2 + x_2)\, \phi^s_{K^*}(\bar{x}_2) + x_2 \phi^t_{K^*}(\bar{x}_2) \right] \phi_K^P(\bar{x}_3) \big\} \\
& \qquad \cdot E^{(q)}(t_q, l^2) \cdot h_e(x_2, x_1, b_2, b_1) \\
& + \big\{ -2 r_{K^*} \phi^s_{K^*}(\bar{x}_2) \phi_K^A(\bar{x}_3) + 4 r_{K^*} r_K \phi^s_{K^*}(\bar{x}_2) \phi_K^P(\bar{x}_3) \big\} \\
& \qquad \cdot E^{(q)}(t'_q, l'^2) h_e(x_1, x_2, b_1, b_2) \Big\}
\end{aligned} \tag{13}
$$

—The evolution factors:

$$E^{(q)}(t,l^2) = C^{(q)}(t,l^2)\,\alpha_s^2(t)\cdot \exp\left[-S_{ab}\right], \tag{14}$$

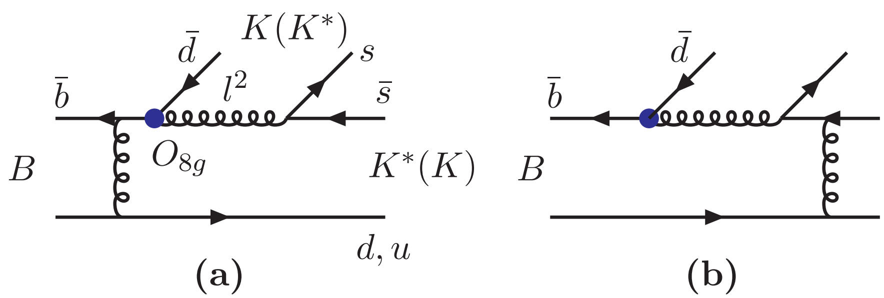
with the Sudakov factor S_{ab};

—The hard function $h_e(x_1,x_2,b_1,b_2)$, coming from the Fourier transformations of hard kernal $H(x_i,b_i)$,

$$\begin{aligned} h_e(x_1,x_2,b_1,b_2) &= K_0\left(\sqrt{x_1x_2}m_Bb_1\right)\left[\theta(b_1-b_2)K_0\left(\sqrt{x_2}m_Bb_1\right)I_0\left(\sqrt{x_2}m_Bb_2\right)\right. \\ &\quad \left.+\theta(b_2-b_1)K_0\left(\sqrt{x_2}m_Bb_2\right)I_0\left(\sqrt{x_2}m_Bb_1\right)\right]S_t(x_2), \end{aligned} \tag{15}$$

$$\begin{aligned} t_q &= \max(\sqrt{x_2}m_B, \sqrt{x_1x_2}m_B, \sqrt{(1-x_2)x_3}m_B, 1/b_1, 1/b_2);\,, \\ t_q' &= \max(\sqrt{x_1}m_B, \sqrt{x_1x_2}m_B, \sqrt{|x_3-x_1|}m_B, 1/b_1, 1/b_2). \end{aligned} \tag{16}$$

Chromo-magnetic penguin contributions



♣ There are 9 relevant Feynman diagrams [Mishima, Sanda, PTP 110 (2003)549], but these two dominate.

♣ O_{8g} contribute at NLO level. For $b \to dg$ transition, $\mathcal{H}_{eff}^{cmp}$ is,

$$\mathcal{H}_{eff}^{cmp} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* \, C_{8g}^{eff} \, O_{8g}, \tag{17}$$

$$O_{8g} = \frac{g_s}{8\pi^2} m_b \, \overline{d}_i \sigma^{\mu\nu}(1+\gamma_5) T^a_{ij} G^a_{\mu\nu} b_j, \quad (18)$$

♣ The total CMP contribution to $B \to KK^*$ decays, for example, can be written as

$$M^{(cmp)}_{KK^*} = \; < K^* K | \mathcal{H}^{cmp}_{eff} | B >= V_{tb} V^*_{td} \left[M^{(g)}_{K^*K} + M^{(g)}_{KK^*} \right], \quad (19)$$

$$\begin{aligned} M^{(g)}_{K^*K} = \; & \frac{4}{\sqrt{3}} G_F C_F^2 m_B^6 \int_0^1 dx_1 dx_2 \, dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \, \phi_B(x_1, b_1) \\ & \cdot \{ \{ -(1-x_2) \left[2\phi_{K^*}(\bar{x}_2) - r_{K^*} \left[3\phi^s_{K^*}(\bar{x}_2) - \phi^t_{K^*}(\bar{x}_2) \right] \right. \\ & \left. - r_{K^*} x_2 \left[\phi^s_{K^*}(\bar{x}_2) + \phi^t_{K^*}(\bar{x}_2) \right] \right] \phi^A_K(\bar{x}_3) \\ & + r_K (1+x_2) \, x_3 \cdot \phi_{K^*}(\bar{x}_2) \left[3\phi^P_K(\bar{x}_3) + \phi^T_K(\bar{x}_3) \right] \end{aligned}$$

$$
\begin{aligned}
&-r_{K^*}r_K\,(1-x_2)\left[\phi^s_{K^*}(\bar{x}_2)+\phi^t_{K^*}(\bar{x}_2)\right]\left[3\phi^P_K(\bar{x}_3)-\phi^T_K(\bar{x}_3)\right] \\
&-r_{K^*}r_K x_3\,(1-2x_2)\left[\phi^s_{K^*}(\bar{x}_2)-\phi^t_{K^*}(\bar{x}_2)\right]\left[3\phi^P_K(\bar{x}_3)+\phi^T_K(\bar{x}_3)\right]\Big\} \\
&\quad\cdot E_g(t_q)h_g(A,B,C,b_1,b_2,b_3,x_2) \\
&+\Big\{4r_{K^*}\phi^s_{K^*}(\bar{x}_2)\phi^A_K(\bar{x}_3)-2r_{K^*}r_K x_3\phi^s_{K^*}(\bar{x}_2)\left[3\phi^P_K(\bar{x}_3)+\phi^T_K(\bar{x}_3)\right]\Big\} \\
&\quad\cdot E_g(t'_q)h_g(A',B',C',b_2,b_1,b_3,x_1)\Big\}\,, \qquad (20)
\end{aligned}
$$

for the case of $B \to K^*$ transition.

NLO pQCD predictions for BR's

Table 5: The NLO pQCD predictions for $Br(B \to K K^*)$ (10^{-7}).

Mode	LO	+VC	+QL	+MP	NLO	Data	QCDF
$B^+ \to K^+ \overline{K}^{*0}$	4.2	5.3	5.8	3.1	3.2	< 11	$3.0^{+6.0}_{-2.5}$
$B^+ \to K^{*+} \overline{K}^0$	2.0	2.7	2.3	1.6	2.1		$3.0^{+7.2}_{-2.7}$
$B^0 \to K^0 \overline{K}^{*0}$	2.1	3.0	2.9	1.8	2.4	–	$2.6^{+2.8}_{-2.0}$
$B^0 \to \overline{K}^0 K^{*0}$	6.4	6.9	8.0	4.3	4.9	–	$2.9^{+7.3}_{-2.7}$
$B^0 \to f_1 + \bar{f}_1$	13.7	14.0	15.2	6.7	8.5	< 19	
$B^0 \to K^+ \overline{K}^{*-}$	1.1	–	–	–	0.83		$0.14^{+1.07}_{-0.14}$
$B^0 \to K^- \overline{K}^{*+}$	0.41	–	–	–	0.17		$0.14^{+1.07}_{-0.14}$
$B^0 \to f_2 + \bar{f}_2$	2.7	–	–	–	1.3		

NLO pQCD predictions for BR's

Table 6: The NLO pQCD predictions for $Br(B \to K\eta^{(\prime)})$ (10^{-6}).

Mode	LO	+VC	+QL	+MP	NLO	Data
$B^+ \to K^+\eta$	4.7	5.3	5.8	3.1	3.2 ± 1.1	2.7 ± 0.3
$B^+ \to K^+\eta'$	30.2	2.7	2.3	1.6	51.0 ± 18	70.2 ± 2.5
$B^0 \to K^0\eta$	3.2	3.0	2.9	1.8	2.1 ± 0.7	< 1.9
$B^0 \to K^0\eta'$	31.3	6.9	8.0	4.3	50.3 ± 16	64.9 ± 3.1
$B^+ \to K^{*+}\eta$	4.8	8.4	6.6	8.2	11.2 ± 3.5	15.9 ± 1.0
$B^+ \to K^{*+}\eta'$	6.1	1.4	0.8	4.3	1.7 ± 0.7	3.8 ± 1.2
$B^0 \to K^{*0}\eta$	4.7	7.5	5.0	8.0	10.5 ± 4.0	19.3 ± 1.6
$B^0 \to K^{*0}\eta'$	3.9	0.84	0.44	3.0	0.6 ± 0.3	4.9 ± 2.1

NLO pQCD predictions for BR's

Table 7: The NLO pQCD predictions for $Br(B \to (\rho, \phi, \omega)\eta^{(\prime)})$ (10^{-6}).

Mode	LO	+VC	+QL	+MP	+NLO	Data
$B^\pm \to \rho^\pm \eta$	6.9	6.8	7.5	7.2	6.7	5.4 ± 1.2
$B^\pm \to \rho^\pm \eta'$	5.2	4.6	4.9	4.7	4.6	$9.1^{+3.7}_{-2.8}$
$B^0 \to \rho^0 \eta$	0.08	0.19	0.16	0.12	0.13	< 1.5
$B^0 \to \rho^0 \eta'$	0.05	0.13	0.06	0.04	0.10	< 1.26
$B^0 \to \omega \eta$	0.22	0.67	0.33	0.25	0.71	< 1.9
$B^0 \to \omega \eta'$	0.12	0.52	0.19	0.15	0.55	< 2.2
$B^0 \to \phi \eta$	0.001	0.011	–	–	0.011	< 0.6
$B^0 \to \phi \eta'$	0.096	0.017	–	–	0.017	< 0.5

NLO pQCD predictions for BR's

Table 8: The NLO pQCD predictions for $Br(B \to \pi\eta^{(\prime)})$ (10^{-6}).

Mode	LO	LO_B	+VC	+QL	+MP	+NLO	Data
$B^{\pm} \to \pi^{\pm}\eta$	2.3	2.5	2.4	2.6	2.4	3.0 ± 1.3	4.4 ± 0.4
$B^{\pm} \to \pi^{\pm}\eta'$	1.3	1.6	1.7	1.6	1.5	2.0 ± 0.8	$2.6^{+0.6}_{-0.5}$
$B^0 \to \pi^0\eta$	0.12	0.14	0.15	0.13	0.14	0.15 ± 0.05	< 1.3
$B^0 \to \pi^0\eta'$	0.03	0.06	0.12	0.07	0.06	0.13 ± 0.06	$1.5^{+0.7}_{-0.6}$

♣ For $(\rho^+, \pi^+)\eta^{(\prime)}$ decays, "T" and "P" dominate! For $(\rho, \pi^0, \omega)\eta^{(\prime)}$ decays, "P" dominate! For $\phi\eta^{(\prime)}$ decays, "Annihilation" only!

♣ The NLO pQCD predictions for CP-violating asymmetries do not show here.

5. Sources of Uncertainties

♣ Higher order corrections. Systematic NLO calculations are clearly needed. NLO contributions through hard-spectator diagrams.

♣ The DAs of heavy and light mesons, the values of Gegenbauer moments; $a^{\pi}_{1,2,4}, \quad a^{K}_{1,2,4}, \quad \omega_b, \quad \eta_3, \ldots.$

♣ The choice of cut-off scale μ_0, the chiral mass $m_0^{\pi}, m_0^{K}, \cdots$.

♣ Mixing schemes of $\eta - \eta'$ system: "singlet-octet" or "quark-flavor". The possible gluonic component of η'.

♣ CKM elements, light quark masses!

6. Two controversial points

♣ Beneke's comment[BEAUTY-2006]: k_T-factorization in pQCD makes integrals convergent, but does not implies that soft contribution can be neglected.

Our Opinion:

— $B \to M_1 M_2$ decays are dominated by hard gluon exchanges. The soft-part absorbed into wave function.

—The pQCD predictions for the values of all relevant form factors agree well with the measured ones, or with those from QCD sum-rules.

— There is no end-point singularity when one uses the $\mathbf{k}_T$ factorization;

— The Sudakov factor can suppress the long-distance part effectively.

♣ Beneke's comment: Wilson coefficients are evaluated at scales down to 0.5 GeV. This is conceptually incorrect. Running stops m_b.

Our opinion:

— The values of $C_{3,4,5,6}(\mu)$ at $\mu = 0.5$ GeV are about four to seven times larger than those at $\mu = 1.0$ GeV.

— In the region of $\mu \geq 1.0$ GeV, however, the $\mu-$dependence of all Wilson coefficients become relatively weak. It is reasonable for us to choose $\mu_0 = 1.0$ GeV as the lower cut-off of the hard scale.

— The pQCD predictions for $Br(B \to KK^*)$ are relatively stable against the variation of μ_0 for $\mu_0 \geq 1.0$ GeV.

7. Summary

♣ For $B/B_s \to M_1 M_2$ decays, the pQCD predictions for BRs generally agree well with the data or the QCDF predictions.

♣ Partial inclusion of the NLO contributions can improve the LO pQCD predictions for BR's, effectively.

♣ For $B^0/B^{\pm}$ decays, the pQCD predictions of the CP-violating asymmetries are generally larger than the QCDF ones.

♣ For $B \to K\pi$ decays, the NLO pQCD predictions of CPV agree with the data!

♣ The contribution of possible gluonic component of $\eta^{(\prime)}$ could be small in size.

♣ The theoretical uncertainties are still large, NLO calculations needed.

Thanks For Your Attention

Gluonic contribution to $F_{0,1}^{B\to\eta'}$

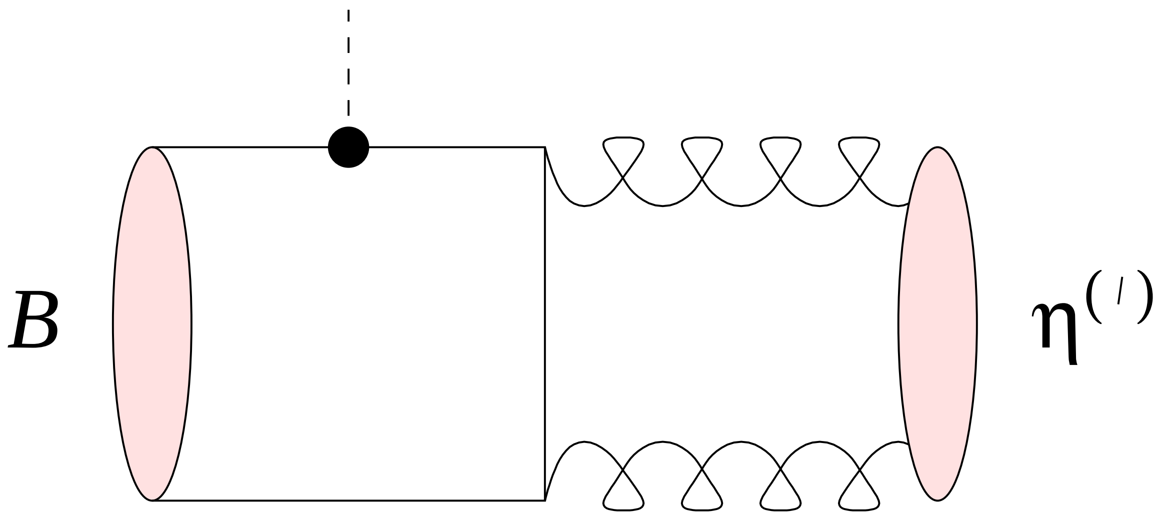
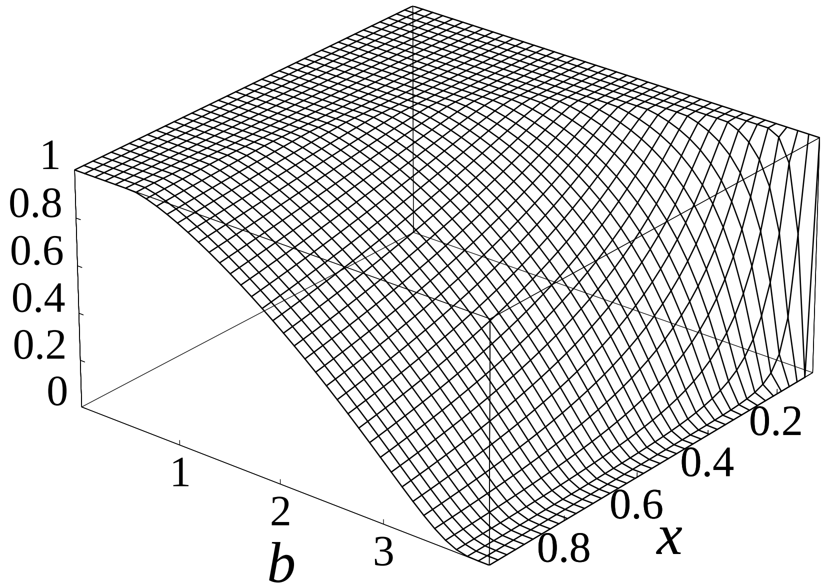


Figure 8: Gluonic contribution to the $B \to \eta^{(\prime)}$ form factors. Another diagram with the two gluons crossed is suppressed. Quoted from Li's paper.

Effects of Sudakov factor $e^{-S(t)}$



♣ In the region of large b, $b \sim b_{max} = 1/\Lambda_{QCD}$, Sudakov factor is very small, close to zero. The long-distance (large b) contributions are therefore suppressed effectively.

FSI, Color Transparency Mechanism

♣ Bjorken [Nucl.Phys. (PS) 11 (1989) 325], to explain why the FSI is small for $B \to M_1 M_2$ decays.

♣ "The argument is based on the space-time evolution of the decay products. At the quark level the decay $b \to u + \pi^-$ begins as a nearly collinear configuration of $b \to u + (\bar{u}d)^-$. The color singlet $\bar{u}d$ pair recoils in the direction opposite to the u. In order that it has good overlap with the final-state pion it has low virtual mass relative to its momentum, of order 2.5 GeV.

♣ It follows that the formation time of the pion will be $\underline{\mathrm{long}}$ because of the relativistic time-dilation. By the time the pion is formed it is several fermis away from the color fields existing in the neighborhood of the original B_d or Λ_b.

♣ And during the time the $\bar{u}d$ system is within those color fields, it is a small color dipole, originating from the point-like, color-singlet weak interaction, and growing only slowly because of the long formation time.

♣ It is therefore arguable that this small dipole will not significantly interact with the spectator system. "

μ_0-dependence of $C_i(\mu)$

Table 9: NLO Wilson coefficients $C_i(\mu)$ for $\mu_0 = 0.5 - 2.0$ GeV.

μ_0	$C_1(\mu)$	$C_2(\mu)$	$C_3(\mu)$	$C_4(\mu)$	$C_5(\mu)$	$C_6(\mu)$
0.5 GeV	-0.9923	1.6537	0.1729	-0.3122	-0.1143	-0.8276
1.0 GeV	-0.5093	1.2790	0.0428	-0.0898	0.0150	-0.1321
1.5 GeV	-0.3773	1.1920	0.0289	-0.0652	0.0153	-0.0856
2.0 GeV	-0.3114	1.1518	0.0230	-0.0541	0.0145	-0.0672

μ_0-dependence of $Br(B \to K K^*)$

Table 10: The pQCD predictions for the branching ratios (in unit of 10^{-7}) and direct CP-violating asymmetries (in unit of 10^{-2}) for the considered $B \to K K^*$ decays, assuming $\mu_0 = 0.5, 1.0, 1.5$ and 2.0 GeV, respectively.

Mode	$\mu_0 = 0.5$	$\mu_0 = 1.0$	$\mu_0 = 1.5$	$\mu_0 = 2.0$
$Br(B^+ \to K^+ \overline{K}^{*0})$	4.7	3.2	2.6	2.1
$Br(B^+ \to K^{*+} \overline{K}^0)$	2.6	2.1	1.3	0.8
$Br(B^0/\overline{B}^0 \to f_1 + \bar{f}_1)$	22.5	8.5	5.0	3.5
$Br(B^0/\overline{B}^0 \to f_2 + \bar{f}_2)$	5.4	1.3	0.78	0.55
$\mathcal{A}_{CP}^{dir}(B^+ \to K^+ \overline{K}^{*0})$	-4.0	-6.9	-7.1	-5.1
$\mathcal{A}_{CP}^{dir}(B^+ \to K^{*+} \overline{K}^0)$	16.8	6.5	-1.5	-5.8