Applications of QCD Sum Rules to Flavour Physics

Alexander Khodjamirian

UNIVERSITÄT SIEGEN

"Advanced Topics on Flavour Physics "

Kavli Institute for Theoretical Physics China, Beijing ,10.07.2008

Outline

- Introducing the method
- Quark mass determination
- Meson decay constants: $f_{B,D}$
- $B \to \pi$ form factors from light-cone sum rules
- Light-cone distribution amplitudes of π,K
- Sum rules with B-meson distribution amplitudes
  $B \to \pi, K, \rho, K^*$, $B \to D^{(*)}$ form factors
- Charmless B-decays:
  estimates of penguin and annihilation amplitudes

QCD sum rules (SVZ)

[M.Shifman, A.Vainshtein and V.Zakharov (1979)]

- Correlator of two quark currents = hadronic sum

$$\int d^4x\, e^{iqx} \langle 0|T\{j_1(x) j_2(0)\}|0\rangle = \sum_h \frac{\langle 0|j_1|h\rangle\langle h|j_2|0\rangle}{m_h^2 - q^2}$$

$$|q^2| \gg \Lambda^2_{QCD} \quad \Downarrow \qquad x \sim 1/\sqrt{|q^2|} \to 0$$

$$\boxed{C_0(q^2, m_q, \alpha_s) + \sum_{d=3,4,..} C_d(q^2, m_q, \alpha_s)\langle 0|O_d|0\rangle}$$

- Local operator product expansion (OPE) (factorization)
  C_d- calculable coeffs, $\langle 0|O_d|0\rangle$- vacuum condensates
- a rigourous dispersion relation, in practice: truncate $\sum\limits_{d=3,4,..}$ and approximate $\sum\limits_h$
- universal input for different j_1, j_2, $h's$

Light-cone sum rules (LCSR)

[I.Balitsky,V.Braun et al (1989); V.Chernyak, I.Zhitnisky (1989)]

- a different type of correlator: $(p^2 = m_H^2)$

$$\int d^4x\, e^{iqx}\langle 0|T\{j_1(x)j_2(0)\}|H(p)\rangle = \sum_h \frac{\langle 0|j_1|h\rangle\langle h|j_2|H\rangle}{m_h^2 - (p-q)^2}$$

$$|q^2| \sim |(p-q)^2| \gg \Lambda^2_{QCD} \qquad \Downarrow \qquad x^2 \to 0$$

$$\boxed{\sum_t C_t(q^2,(p-q)^2,m_q,\alpha_s)\langle 0|O_t(x,0)|H(p)\rangle}$$

- OPE near the light-cone (factorization), C_t- calculable,

$\langle 0|O_t(x,0)|H(p)\rangle$ - light-cone distribution amplitudes (DA's)

Twofold use of QCD sum rules:

I. hadronic sum from experiment
$\Rightarrow$ QCD/OPE parameters:
m_q, condensates, DA's

II. correlator from OPE
$\Rightarrow$ hadronic matrix elements:
$\langle 0|j_1|h\rangle$, $\langle h|j_2|H\rangle$

- applications to flavour physics:

  I. determination of quark masses,
  II. hadron decay constants, form factors $\Rightarrow |V_{CKM}|$

- an introductionary review:

  e.g., [A.K., P. Colangelo, hep-ph/0010175]

Recent applications

- SVZ sum rules
  - quark masses: m_s with 5-loop accuracy
  - B, D decay constants: f_{D_s}, is there a puzzle ?
  - SU(3)-asymmetry (a_1^K) in the kaon DA
- LCSR
  - $B \to \pi$ form factor and $|V_{ub}|$
  - $B \to D^{(*)}$ form factors at large recoil
  - amplitudes of $B \to PP$ decays , $P = \pi, K$

Quark mass determination

- Light quark masses

$$m_q \equiv \overline{m}_q(2\text{ GeV}),\ q = u, d, s$$

- less accurate than the other SM parameters:
  in PDG 2006 $\sim$ 25% accuracy for m_s ;
  compared: $\sim$ 10% for m_c and $\sim$ 2% for m_b
- the reason: $\Lambda_{QCD} \sim m_s \gg m_u, m_d$:
  small influence of $m_{u,d,s}$ on hadronic observables
  (exception: $m_{\pi,K,\eta}$)

Light quark masses

- Chiral Perturbation Theory:

$$R = \frac{m_s}{\hat{m}} = 24.4 \pm 1.5, \quad \mathcal{Q}^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} = (22.7 \pm 0.8)^2$$

[Leutwyler, 1996]

$$\hat{m} = \tfrac{1}{2}(m_u + m_d)$$

$\Rightarrow$ determine m_s, obtain $m_{u,d}$ “for free “:

$$m_d = \frac{m_s}{R}\left(1 + \frac{R-1}{4\mathcal{Q}^2}\right), \quad m_u = \frac{m_s}{R}\left(1 - \frac{R-1}{4\mathcal{Q}^2}\right)$$

“Nonlattice” methods of m_q determination

- based on quark-current correlators and OPE:
  - positivity bounds
  - QCD (SVZ) sum rules
  - Finite-energy sum rules (FESR)
  - inclusive $\tau \to s\bar{u}\nu_\tau$ decays

m_s from QCD sum rules

- Correlator with scalar (pseudoscalar) currents:

$$j_{S(P)} = \partial^\mu \bar{s}\gamma_\mu(\gamma_5)q = (m_s \,{}_{(\mp)}\, m_q)\bar{s}(\gamma_5)q, \qquad (q = u, d)$$

$$\boxed{\Pi^{(P)}(q^2) = i\int d^4x\, e^{iq\cdot x}\langle 0|T\left\{j_P(x)j_P^\dagger(0)\right\}|0\rangle}$$

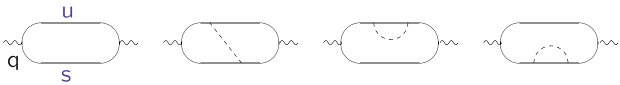
- Dispersion relation (doubly differentiated) for $\Pi^{(P)}(q^2)$ at $Q^2 = -q^2 \gg \Lambda^2_{QCD}$:

$$[\Pi^{(P)''}(q^2)]_{OPE} = 2\int\limits_0^\infty ds \frac{\rho^{(P)}(s)}{(s-q^2)^3},$$

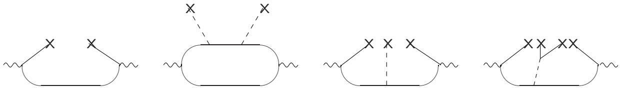
$$\rho^{(P)}(s) = \sum_K \langle 0|j_P|K(q)\rangle\langle K(q)|j_P|0\rangle$$

$\sum_K$ = {kaon $\oplus$ excitations} $\oplus$ quark-hadron duality

Diagrams contributing to OPE



$\oplus O(\alpha_s^2) \oplus O(\alpha_s^3) \oplus O(\alpha_s^4)$



$\langle \bar{q}q \rangle$ $\langle GG \rangle$ $\langle \bar{q}Gq \rangle$ $\langle \bar{q}q \rangle^2$

$\oplus O(\alpha_s)$

$O(\alpha_s^4)$ calculated *[Baikov, Chetyrkin, Kühn (2005)]*

OPE for the pseudoscalar correlator

expansion in $1/(Q)^{d+2}$, $d = 0, 2, 4, 6$

$$[\Pi^{(P)''}(Q^2)]_{OPE} = \frac{3(m_s + m_u)^2}{8\pi^2 Q^2} \left\{ 1 + \sum_{i=1}^{4} C_{0,i} \left(\frac{\alpha_s}{\pi}\right)^i \right.$$
$$\left. -2\frac{m_s^2}{Q^2} \left(1 + \sum_{i=1,2} C_{2,i} \left(\frac{\alpha_s}{\pi}\right)^i \right) + \frac{\{d=4\}}{Q^4} + \frac{\{d=6\}}{Q^6} \right\}$$

$\{d = 4\} \sim \{m_s\langle\bar{q}q\rangle, \langle G^2\rangle, O(m_s^4)\}\,(1 \oplus O(\alpha_s))$
$\{d = 6\} \sim m_s\langle\bar{q}Gq\rangle, \langle\bar{q}q\rangle^2$

- vacuum condensate densities: $\langle ...\rangle = \langle 0|...|0\rangle$

$O_3 = \bar{q}q$, $O_4 = G^{a\mu\nu}G^a_{\mu\nu}$, $O_5 = \bar{q}\sigma_{\mu\nu}(\lambda^a/2)G^{a\mu\nu}q$, $O_6 = \bar{q}\Gamma_a q\bar{q}\Gamma_a q$

Coefficients multiplying $(\alpha_s/\pi)^n$ in $d = 0$ part: ($l_Q = \log Q^2/\mu^2$)

$$C_{0,1} = \frac{11}{3} - 2\,l_Q\,, \quad C_{0,2} = \frac{5071}{144} - \frac{35}{2}\zeta_3 - \frac{139}{6}\,l_Q + \frac{17}{4}\,l_Q^2,$$

$$C_{0,3} = \frac{1995097}{5184} - \frac{\pi^4}{36} - \frac{65869}{216}\zeta_3 + \frac{715}{12}\zeta_5 - \frac{2720}{9}\,l_Q + \frac{475}{4}\zeta_3\,l_Q + \frac{695}{8}\,l_Q^2 - \frac{221}{24}l_Q^3,$$

most recent:

$$\begin{aligned} C_{0,4} = & \quad \frac{2361295759}{497664} - \frac{2915}{10368}\pi^4 - \frac{25214831}{5184}\zeta_3 + \frac{192155}{216}\zeta_3^2 + \frac{59875}{108}\zeta_5 - \frac{625}{48}\zeta_6 \\ - & \quad \frac{52255}{256}\zeta_7 + l_Q\left[-\frac{43647875}{10368} + \frac{1}{18}\pi^4 + \frac{864685}{288}\zeta_3 - \frac{24025}{48}\zeta_5\right] \\ + & \quad l_Q^2\left[\frac{1778273}{1152} - \frac{16785}{32}\zeta_3\right] + l_Q^3\left[-\frac{79333}{288}\right] + l_Q^4\left[\frac{7735}{384}\right], \end{aligned}$$

Hierarchy in α_s and d

- Relative contributions to $[\Pi^{(P)''}(M^2)]_{OPE}$
  (after Borel transformation $Q^2 \to M^2$)

$$r_n^{(d)}(M^2) = \frac{\{\Pi^{(P)''}(M^2)\}^{(d)}_{O(\alpha_s^n)}}{\Pi^{(P)''}(M^2)}$$

$$r_n^{(d=0,2)}(2.5\ \text{GeV}^2) = 52.4\%,\ 28.3\%,\ 14.4\%,\ 4.0\%,\ -0.3\%$$

$$(n = 0, 1, 2, 3, 4)$$

$$r^{(d=4,6)}(2.5\ \text{GeV}^2) = 1.2\%.$$

- power suppressed corrections very small:

uncertainties of vacuum condensate densities inessential

The hadronic sum

- $\{K, K2\pi, K^*\pi, \rho K, ...\}$
  $\rightarrow$ 3-resonance ansatz $\{K, K_1(1460), K_2(1830))\}$

$m_{K_1} = 1460$ MeV, $\Gamma_{K_1} = 260$ MeV;
$m_{K_2} = 1830$ MeV, $\Gamma_{K_2} = 250$ MeV [PDG]

$$\rho^{(P)}_{hadr}(s) = f_K^2 m_K^4 \delta(m_K^2 - s) + \sum_{i=1,2} f_{K_i}^2 m_{K_i}^4 \frac{1}{\pi} \left(\frac{\Gamma_{K_i} m_{K_i}}{(s - m_K^2)^2 + (\Gamma_{K_i} m_{K_i})^2} \right)$$

- decay constants: $\langle 0|j_P|K(q)\rangle = f_K m_K^2$,

$f_K = 159.8$ MeV , $f_{K_1,K_2} \ll f_K$ (ChPT)
fitted combining various moments of sum rules and/or FESR

[Kambor,Maltman '03]

- use of quark-hadron duality for the continuum: negligible contribution of states at $s > m_{K_2}^2$

Quark-hadron duality

- global duality for the hadronic sum (dispersion relation):

strict property based on asymptotic freedom of QCD

$$\sum_h \frac{\langle 0|j_1|h\rangle\langle h|j_2|0\rangle}{m_h^2 - q^2} \equiv \int\limits_{m_{h_0}^2}^{\infty} ds \frac{\rho_{hadr}(s)}{s - q^2} = \frac{1}{\pi} \int\limits_{(m_q+m_{q'})^2}^{\infty} ds \frac{\mathrm{Im}C_0(s)}{s - q^2}$$

h_0 - the lowest hadron with flavour $\{\bar{q}q'\}$, ($j_1 = \bar{q}\Gamma q', j_2 = \bar{q}'\Gamma q$), (e.g., π, K, D, B)

- local duality:

$$\rho_{hadr}(s) \simeq \frac{1}{\pi}\mathrm{Im}C_0(s),$$

an approximation valid at sufficiently large $s \gg m_{h_0}^2$

“Semilocal” duality used in QCD sum rules

$$\frac{\langle 0|j_1|h_0\rangle\langle h_0|j_2|0\rangle}{m_{h_0}^2-q^2}+\sum_{h\neq h_0}\frac{\langle 0|j_1|h\rangle\langle h|j_2|0\rangle}{m_h^2-q^2}$$

$$=\frac{1}{\pi}\int\limits_{(m_{q_1}+m_{q_2})^2}^{s_0} ds\frac{\text{Im}C_0(s)}{s-q^2}+\frac{1}{\pi}\int\limits_{s_0}^{\infty} ds\frac{\text{Im}C_0(s)}{s-q^2}$$

matching the sum of excited $\oplus$ continuum h-states
to the integral over ImC_0
with s_0, the effective threshold

Comments on quark-hadron duality

- "semilocal" duality is a weaker assumption than the local one
- works for channels where the hadronic sum is measured
(J/ψ) or dominated by the lowest state (π, K)
- a systematic uncertainty is introduced in the sum rule
- Borel transformation

$$\boxed{\frac{1}{(m_h^2 - q^2)} \rightarrow exp(-m_h^2/M^2)}$$

suppresses the higer-state contributions to the hadronic sum,
the sum rule less sensitive to the duality approximation

- no standard approach to fix s_0, e.g., calculating
the mass of h_0 from the same sum rule by $d/d(1/M^2)$
- "Borel stability" can be misleading !

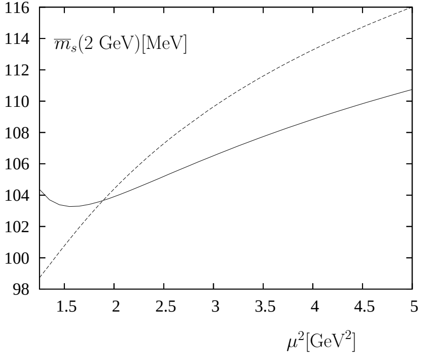
Pseudoscalar sum rule to $O(\alpha_s^4)$

[Chetyrkin, A.K.,(2006)]

- The result:

$$\overline{m}_s(2\text{ GeV}) = \left(105 \pm 6\Big|_{OPE} \pm 7\Big|_{hadr}\right)\text{MeV}\,,$$

- if the $O(\alpha_s^4)$ terms are removed: $\simeq$ 2 MeV increase of the central value
- The scale dependence: solid- $O(\alpha_s^4)$, dashed- $O(\alpha_s^3)$



Recent m_s determinations

Method, accuracy	m_s(2GeV) [MeV]	References
OPE bound, $O(\alpha_s^4)$	> 76	Baikov,Chetyrkin, Kühn '05
QCD SR (P), $O(\alpha_s^4)$	$105 \pm 6 \pm 7$	A.K., Chetyrkin '05
QCD SR (P), $O(\alpha_s^4)$	97^{+11}_{-8}	Jamin, Oller, Pich '06
QCD SR (S), $O(\alpha_s^4)$	88^{+9}_{-7}	Jamin, Oller, Pich '06
FESR (P), $O(\alpha_s^4)$	102 ± 8	Dominguez et al.'08
Lattice QCD, $2 \oplus 1$	87 ± 4 ± 4	HPQCD'06 Mason et al.
Lattice QCD, $2 \oplus 1$	$90 \pm 5 \pm 4$	MILC '06 Bernard et al.
Lattice QCD,$2 \oplus 1$	$91.1^{+14.6}_{-6.2}$	CP-PACS/JLQCD '07 Ishikawa et al.
Lattice QCD,$2 \oplus 1$	107.3 ± 4.4 $\pm 9.7 \pm 4.9$	RBC /UKQCD '08 Allton et al,

Comments

- all $O(\alpha_s^4)$ sum rule intervals agree with
the $2 \oplus 1$ lattice determinations ,

uncertainty in m_s smaller than in PDG'06

- all m_s determinations obey the OPE bound,
- P, S sum rules:

no further need for improvement of OPE for $\Pi^{(P,S)}(q^2)$

- The hadronic sum in P -channel:
radially excited kaon ($J^P = 0^-$) states
(resonances in $K\pi\pi$ and $K^*\pi, \rho K$),
accessible, e.g. in $\tau \to K_1 \nu_\tau$, $D \to K_1 l \nu_l$, $B \to K_{1,2} D^{(*)}$

- scalar sum rules, a more complicated hadronic sum:
nonres. $K\pi$ ($J^P = 0^+$) states important

c, b-quark masses from sum rules

- recent determinations:

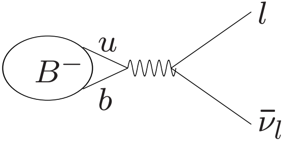
first moments of quarkonium sum rules (relativistic), $O(\alpha_s^3)$ accuracy achieved:

$\bar{m}_c(\bar{m}_c)$ [GeV]	$\bar{m}_b(\bar{m}_b)$ [GeV]	Reference
1.286 ± 0.013	4.164 ± 0.025	Kühn,Steinhauser, Sturm '06
1.295 ± 0.015	4.205 ± 0.058	Boughezal,Czakon, Schutzmeier '06

- more accurate values of quark masses $\Rightarrow$
  relations between quark/lepton masses
  in hypothetical flavour scenarios

B,D-meson decay constants

$B, D_{(s)} \to l\bar{\nu}_l \quad (l = e, \mu, \tau)$, recent experimental progress



the hadronic matrix element:

$$m_b \langle 0|\bar{u} i\gamma_5 b|B^-\rangle = f_B m_B^2$$

- $f_{B_{(s)}}$ will be needed for more complicated hadronic matrix elements
- recent $CLEO_c$ measurement of f_{D_s} has created some tension with SM

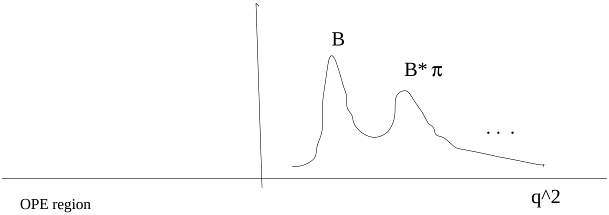
QCD sum rule for f_B

Correlation function of two $j^{(B)}(x) = m_b \bar{u}(x) i\gamma_5 b(x)$ currents:

$$\Pi(q^2) = i \int d^4x e^{iqx} \langle 0 \mid T\{j^{(B)}(x) j^{(B)}(0)\} \mid 0\rangle$$

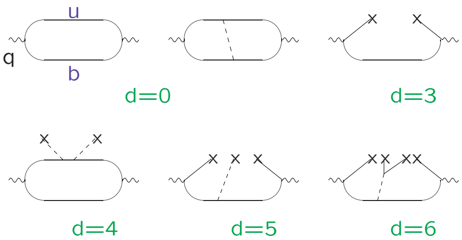
$$= \sum_{h=B,B^*\pi,\dots} \frac{\langle 0 \mid j^{(B)} | h\rangle\langle h | j^{(B)} \mid 0\rangle}{m_h^2 - q^2}$$

A dual object



hadronic sum

OPE diagrams



- different coefficients C_d miltiplied with the same universal condensates
- quark and quark-gluon condensate terms enhanced $\sim m_b$

The sum rule result for f_B

- inputs: $\bar{m}_b$ (see quark mass determination),
$\langle \bar{q}q \rangle$ (PCAC), $\langle G^2 \rangle$ (J/ψ SR) $\oplus$ quark-hadron duality

- the sum rule result with $O(\alpha_s^2)$ accuracy,
at $\bar{m}_b(\bar{m}_b) = 4.21 \pm 0.05$ GeV :
$f_B = 210 \pm 19$ MeV, $f_{B_s} = 244 \pm 21$ MeV

[M.Jamin,B.O.Lange(2001)]

$f_B = 206 \pm 20$ MeV, (HQET)

[A.Penin, M.Steinhauser (2001)]

agree (within still large errors) with experiment on $B \to \tau \nu_l$ (V_{ub}- PDG average)

and with the lattice QCD determinations of $f_{B_{(s)}}$

f_D, f_{D_s}

- Experiment $f_D = 223 \pm 17$ MeV, $f_{D_s} = 275 \pm 10$ MeV

  [J.Rosner, S.Stone , for PDG08]

$f_D = 205 \pm 8.5 \pm 2.5$ MeV, $f_{D_s} = 267.9 \pm 8.2 \pm 3.9$ MeV

[S.Stone talk at FPCP (2008), 0806.3921[hep-ex]]

- QCD sum rule predictions:

$f_D = 195 \pm 20$ MeV

[A.Penin,M.Steinhauser '01] ($O(\alpha_s^2)$ QCD SR in HQET)

no definite interval

[M.Jamin, B.Lange '01] ($O(\alpha_s^2)$ full QCD, MS)

$f_D = 203 \pm 20$ MeV , $f_{D_s} = 235 \pm 24$ MeV *[S.Narison '02]*

$f_{D_s} > f_D$ predicted but lower than exp.

- the latest lattice QCD results :

$f_D = 207 \pm 4$ MeV , $f_{D_s} = 241 \pm 3$ MeV

[Follana et al.HPQCD and UKQCD,2007]

a rigorous upper bound for $f_{D_{(s)}}$

- from the same correlator/OPE :

$$f_D^2 m_D^4 e^{-m_D^2/M^2} + ... = \Pi(M^2; m_c, m_s, \alpha_s, \text{cond.}, \ \mu,)$$

- the correlator has a positive definite spectral density

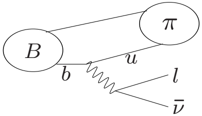
$$\Rightarrow \ f_D < \sqrt{\Pi(M^2)/(m_D^4 e^{-m_D^2/M^2})}$$

- preliminary result without $O(\alpha_s^2)$, M and μ optimally small ($\sim$ 1 GeV)

$f_D < 230$ MeV , $f_{D_s} < 260$ MeV

- exp. result for f_{D_s} looks indeed unexpectedly large ...

$B \to \pi$ form factors



$\Rightarrow |V_{ub}|$

- $B \to \pi l \bar{\nu}$ ($l = e, \mu, \tau$), measured at CLEO, Belle, BaBar

$$\langle \pi(p) | \bar{u} \gamma_\mu b | B(p+q) \rangle = f^+_{B\pi}(q^2)(2p_\mu + q_\mu) + f^-_{B\pi}(q^2) q_\mu$$

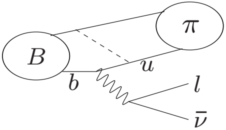
$$f^0_{B\pi}(q^2) = f^+_{B\pi}(q^2) + \frac{q^2}{m_B^2 - m_\pi^2} f^-_{B\pi}(q^2)$$

- semileptonic region $0 < q^2 < (m_B - m_\pi)^2$

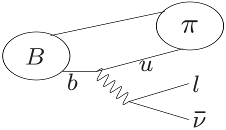
$$\frac{d\Gamma(B \to \pi l \nu_l)}{dq^2} \sim |V_{ub}| |f^+_{B\pi}(q^2)|^2 \quad (l = e, \mu)$$

only large q^2 accessible on the lattice

The factorization problem



"hard", factoriz.



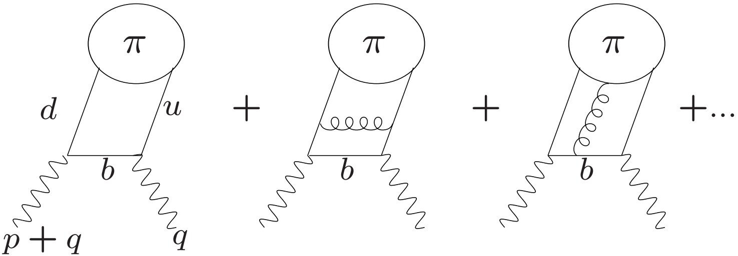
"soft", nonfact.

$$f_{B\pi}(q^2) \sim \alpha_s(\mu) \int d\omega du \phi_+^B(\omega,\mu) T_h(q^2,\omega,u,\mu)\varphi_\pi(u,\mu) + f_{B\pi}^{soft}(q^2)$$

T_h-hard-scattering amplitude,
$\varphi_{B,\pi}$ -light-cone DA's of B and π

how important is $f_{B\pi}^{soft}(q^2)$?

Light-Cone Sum Rules for $B \to \pi$: the correlator



$q^2, (p+q)^2 \sim m_b^2 - m_b\chi$, b-quark highly virtual $\Rightarrow$ $x^2 \sim 0$

$$F_\lambda(q,p) = i \int d^4x e^{iqx} \langle \pi(p) \mid T\{\bar{u}(x)\gamma_\lambda b(x), \bar{b}(0) i\gamma_5 d(0)\} \mid 0 \rangle$$

- an intermediate scale $\Lambda_{QCD} \ll \chi \ll m_b$

OPE near the light-cone

$$F(q,p) = i\int d^4x e^{iqx}\Bigg\{ \Big[S_0(x^2,m_b^2,\mu) + \alpha_s S_1(x^2,m_b^2,\mu)\Big]$$

$$\otimes\langle\pi(p)\mid\bar{u}(x)\Gamma d(0)\,|0\rangle|_\mu$$

$$+\int_0^1 dv\ \tilde{S}(x^2,m_b^2,\mu,v)\otimes\langle\pi(p)\mid\bar{u}(x)G(vx)\tilde{\Gamma}d(0)\}\mid 0\rangle|_\mu\Bigg\} + ...$$

- $S_{0,1}$, $\tilde{S}$ - perturbative amplitudes, (b-quark propagators)
- vacuum-pion matrix elements - expanded near $x^2 = 0$

$\Rightarrow$ universal distribution amplitudes of π :

$$\langle\pi(q)|\bar{u}(x)[x,0]\gamma_\mu\gamma_5 d(0)|0\rangle_{x^2=0} = -iq_\mu f_\pi\int_0^1 du\, e^{iuqx}\varphi_\pi(u) + O(x^2)\,.$$

- the expansion goes over twists
- terms $\sim\tilde{S}$ suppressed by powers of $1/\mu^2$; $\mu^2\sim m_b\Lambda$

The OPE result

$$F(q^2,(p+q)^2) = \sum_{t=2,3,4,..} \int du\, T^{(t)}(q^2,(p+q)^2,m_b^2,\alpha_s,u,\mu)\, \varphi_\pi^{(t)}(u,\mu)$$

hard scattering amplitudes $\otimes$ pion light-cone DA

- LO twist 2,3,4 $q\bar{q}$ and $\bar{q}qG$ terms:

  [V.Belyaev, A.K., R.Rückl (1993); V.Braun, V.Belyaev, A.K., R.Rückl (1996)]

-NLO $O(\alpha_s)$ twist 2, (collinear factorization)

  [A.K., R.Rückl, S.Weinzierl, O. Yakovlev (1997); E.Bagan, P.Ball, V.Braun (1997);]

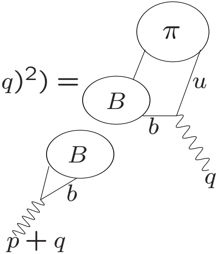
-NLO $O(\alpha_s)$ twist 3 (coll.factorization for asympt. DA)

  [P. Ball, R. Zwicky (2001); G.Duplancic,A.K.,B.Melic, Th.Mannel,N.Offen (2007)]

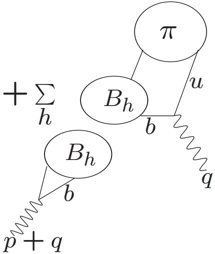
Derivation of LCSR

- Hadronic dispersion relation in the variable $(p+q)^2$:

$F(q^2,(p+q)^2) =$



$+ \sum_h$



$f_B f^{+}_{B\pi}(q^2)$ $\qquad$ $\sum_{B_h} \to duality\ (s_0^B)$

($q^2 \ll m_b^2$ fixed)

Derivation of LCSR

- matching at $\langle -(p+q)^2\rangle \sim \mu^2 \sim m_b\chi$ and using duality

$$[F((p+q)^2,q^2)]_{OPE} = \frac{m_B^2 f_B f^+_{B\pi}(q^2)}{m_B^2-(p+q)^2} + \int\limits_{s_0^B}^{\infty} ds \frac{[\mathrm{Im} F(s,q^2)]_{OPE}}{s-(p+q)^2}$$

- inputs: $\overline{m}_b$, α_s, $\varphi_\pi^{(t)}(u)$, t=2,3,4;
  f_B - determined from two-point (SVZ) sum rule;
  optimal interval of M^2, μ, s_0^B
- uncertainties due to:
  - variation of (universal) input parameters,
  - quark-hadron duality

(suppressed with Borel transformation, controlled by the m_B calculation)

- LCSR contains *both* “soft” and “hard” contributions to $f_{B\pi}$
- initial light-cone OPE $\rightarrow 1/(m_b\chi)$ expansion,

the resulting form factor $1/m_b$ expansion is less trivial,

the method is used at finite m_b

Distribution amplitudes (DA's) of the pion

- a topical problem for all factorization approaches
- twist 2 DA: normalized with f_π,
  expansion in Gegenbauer polynomials

$$\varphi_\pi(u,\mu) = 6u(1-u)\left[1 + \sum_{n=2,4,..} a_n^\pi(\mu) C_n^{3/2}(2u-1)\right],$$

$$a_{2n}^\pi(\mu) \sim [Log(\mu/\Lambda_{QCD})]^{-\gamma_{2n}} \to 0 \quad \text{at} \;\; \mu \to \infty$$

[Efremov-Radyushkin-Brodsky-Lepage evolution]

Gegenbauer moments

- the most important parameters: $a^{\pi}_{2,4}(\mu_0)$, determined from:
  - matching exp. pion form factors to LCSR,
  - two-point QCD sum rules,
  - lattice QCD
- $a_2^{\pi} = 0.25 \pm 0.15$ (average. of recent determinations)

$a_2^{\pi} + a_4^{\pi} = 0.1 \pm 0.1$ (pion-photon form factor)

- remaining tw 3,4 DA parameters:

normalization constants and first moments,
determined mainly from two-point sum rules

[P. Ball, V.Braun, A.Lenz (2006)]

Light-cone DA of the kaon (twist-2)

$$\langle K^-(q)|\bar{s}(z)\gamma_\mu\gamma_5\,[z,-z]\,u(-z)|0\rangle_{z^2\to 0}$$
$$= -iq_\mu f_K \int_0^1 du\; e^{iuq\cdot z - i\bar{u}q\cdot z}\varphi_K(u,\mu)\,,$$

- Gegenbauer expansion

$$\varphi_K(u,\mu) = 6u\bar{u}\Big\{1 + a_1^K(\mu)C_1^{3/2}(u-\bar{u}) + a_2^K(\mu)C_2^{3/2}(u-\bar{u}) + ...\Big\},$$

- note that $a_1^\pi \sim 0$ (isospin synmmetry)
- does the valence s-quark in the kaon

have a larger average momentum than the antiquark?

intuitively (quark model) $\rightarrow$ “yes”

a_1^K and $SU(3)_{fl}$ violation in DA's

- $a_1^K \neq 0 \sim (m_s - m_{u,d})$ $\quad (f_K/f_\pi\ ,\ a_2^K/a_2^\pi)$
- a_1^K influences $SU(3)_{fl}$ relations between form factors and/or charmless B decay amplitudes (QCDF, LCSR)
- related to the local hadronic matrix element

$$\langle K^-(q)|\bar{s}\gamma_\nu\gamma_5 i\overleftrightarrow{D}_\lambda u|0\rangle = -iq_\nu q_\lambda f_K \frac{3}{5} a_1^K\,,$$

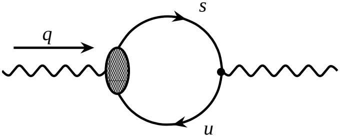
- multiplicative renormalization

$$a_1^K(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{\gamma_0}{\beta_0}} \left(\frac{\beta_0 + \beta_1(\alpha_s(\mu_0)/\pi)}{\beta_0 + \beta_1(\alpha_s(\mu)/\pi)}\right)^{\left(\frac{\gamma_0}{\beta_0} - \frac{\gamma_1}{\beta_1}\right)} a_1^K(\mu_0)\,,$$

Determination of a_1^K at NNLO level

[Chetyrkin, A.K., A.Pivovarov (2007)]

- 2-point (SVZ) sum rule for a_1^K



- The correlator

$$\Pi_{\mu\nu\lambda}(q) = i\int d^4x\, e^{iq\cdot x}\langle 0|T\left\{\bar{u}(x)\gamma_\mu\gamma_5 s(x), \bar{s}(0)\gamma_\nu\gamma_5 i \overleftrightarrow{D}_\lambda u(0)\right\}|0\rangle$$
$$= q_\mu q_\nu q_\lambda \Pi(q^2) + \dots,$$

- Expansion in powers of $1/Q^2$

$$\Pi(Q^2,\mu) = \frac{\mathcal{A}_2(Q^2,\mu)}{Q^2} + \frac{\mathcal{A}_4(Q^2,\mu)}{Q^4} + \frac{\mathcal{A}_6(Q^2,\mu)}{Q^6} + \dots,$$

Expansion in α_s, m_s^2/Q^2

$$
\begin{aligned}
\mathcal{A}_2(Q^2,\mu) &= \frac{m_s^2}{4\pi^2}\Bigg(1+\frac{\alpha_s}{\pi}\left[\frac{26}{9}+\frac{10}{9}l_Q\right] \\
&+ \left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{366659}{11664}-\frac{29}{9}\zeta(3)+\frac{14449}{972}l_Q+\frac{605}{324}l_Q^2\right] \\
&+ 3\frac{m_s^2}{Q^2}\left(\frac{5}{2}+l_Q\right)\Bigg).
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_4(Q^2,\mu) &= -m_s\langle\bar{s}s\rangle\Bigg(1-\frac{\alpha_s}{\pi}\left[\frac{112}{27}+\frac{8}{9}l_Q\right] \\
&- \left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{28135}{1458}-4\zeta(3)+\frac{218}{27}l_Q+\frac{49}{81}l_Q^2\right]+2\frac{m_s^2}{Q^2}\Bigg) \\
&- m_s\langle\bar{u}u\rangle\left(\frac{4\alpha_s}{9\pi}+\left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{59}{54}+\frac{49}{81}l_Q\right]\right),
\end{aligned}
$$

$\langle\bar{q}q\rangle \equiv \langle 0|\bar{q}q|0\rangle$, ($q = s, u$) quark-condensate,
$\mathcal{A}_6$ contains $d = 4, 5, 6$ condensates

The result

$$a_1^K(1\ \text{GeV}) = 0.10 \pm 0.04$$

with evolution: $a_1^K(2\ \text{GeV}) = 0.08 \pm 0.04$,

- NLO corrections reshuffle OPE,
  $O(m_s^2)$ loop becomes important
- old sum rule estimates: ($\langle \bar{q}q \rangle$ terms dominant)
  $$a_1^K(1\text{GeV}) = 0.06 \pm 0.03$$
- comparison with lattice QCD results:

$a_1^K(2\ \text{GeV}) = 0.0453 \pm 0.0009 \pm 0.0029$,

[V.M.Braun et al., [QCDSF/UKQCD] (2006)]

$a_1^K(2\ \text{GeV}) = 0.048 \pm 0.003$,

[M. A. Donnellan et al. (2007)]

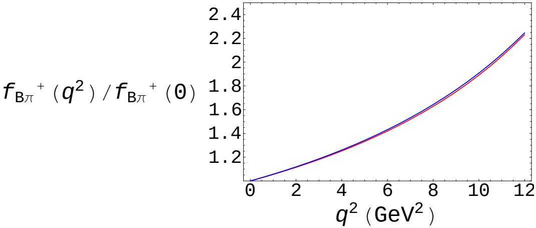
with a linear extrapolation in m_s , $O(m_s^2)$ effect?

Recent update of LCSR for $f^+_{B\pi}(q^2)$

[G. Duplancić, A.K., Th. Mannel, B. Melić, N. Offen, arXiv:0801.1796 [hep-ph],JHEP]

- $\overline{MS}$ b quark mass used
- twist-3 $O(\alpha_s)$ corrections recalculated
- $\varphi_\pi(u)$, Gegenbauer moments at low scale 1 GeV:
- fitting the q^2 dependence to the measured slope:

$a_2^\pi(1\text{GeV}) = 0.16 \pm 0.01$; $a_4^\pi(1\text{GeV}) = 0.04 \mp 0.01$



plot: LCSR vs BK parametrization of the BABAR data (almost indistinguishable):

we “trade” the $q^2 \neq 0$ LCSR calculation for the accuracy of the $f^+_{B\pi}(0)$ prediction

Extracting $|V_{ub}|$

$$f^+_{B\pi}(0) =$$

$$0.262 \pm [0.005]_{fit} \pm [0.002]_{m_b} \begin{bmatrix} +0.03 \\ -0.02 \end{bmatrix}_{m_q} \pm [0.002]_M \pm [0.001]_\mu \pm ...$$

combining all individual uncertainties in quadrature:

$$\boxed{f^+_{B\pi}(0) = 0.26^{+0.04}_{-0.03}}$$

- using $|V_{ub} f^+(0)|$ from P. Ball's fit of BaBar data:

$$|V_{ub}| = \left(3.5[\pm 0.4]_{th} \pm [0.2]_{shape} \pm [0.1]_{BR} \right) \times 10^{-3} \,,$$

- most recent LCSR result:

  (with one-loop pole mass $m_b = 4.8 \pm 0.1$ GeV)

$f^+_{B\pi}(0) = 0.258 \pm 0.031$ *[P.Ball, R.Zwicky(2004)]*

Recent $|V_{ub}|$ determinations from $B \to \pi l \nu_l$

[ref.]	$f^+_{B\pi}(q^2)$ calculation	$f^+_{B\pi}(q^2)$ input	$\lvert V_{ub}\rvert \times 10^3$
Okamoto et al.	lattice ($n_f = 3$)	-	3.78±0.25±0.52
HPQCD	lattice ($n_f = 3$)	-	3.55±0.25±0.50
Arnesen et al.	-	lattice⊕SCET	$3.54 \pm 0.17 \pm 0.44$
Becher,Hill	-	lattice	$3.7 \pm 0.2 \pm 0.1$
Flynn et al	-	lattice ⊕ LCSR	$3.47 \pm 0.29 \pm 0.03$
Ball, Zwicky	LCSR	-	$3.5 \pm 0.4 \pm 0.1$
this work	LCSR	-	$3.5 \pm 0.4 \pm 0.2 \pm 0.1$

- apparently no tension anymore between exclusive and (updated in 2007) inclusive $|V_{ub}|$ determinations

$B_{(s)} \to K$ form factors: an update

- including m_s in OPE $\to$ kaon DA's (a_1^K)
  [G. Duplancić, B. Melić , hep-ph 0806]
- ratios (some uncertainties cancel)

$$\frac{f^+_{BK}(0)}{f^+_{B\pi}(0)} = 1.38^{+0.11}_{-0.10} \quad \frac{f^+_{B_sK}(0)}{f^+_{B\pi}(0)} = 1.15^{+0.17}_{-0.09}$$

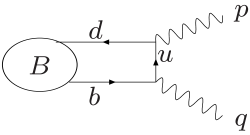
- relevant for $B \to Kll$,
  $SU(3)_{fl}$ relations for $B \to hh$ amplitudes:

$$\xi = \frac{f_K}{f_\pi}\frac{f^+_{B\pi}(m_K^2)}{f^+_{B_sK}(m_\pi^2)}\frac{m_B^2 - m_\pi^2}{m_{B_s}^2 - m_K^2} = 1.01^{+0.07}_{-0.15}.$$

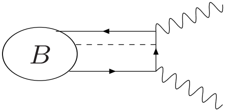
- close to previous LCSR estimates of $SU(3)_{fl}$ violation

LCSR with B-meson distribution amplitudes

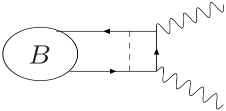
[A.K., T. Mannel, N.Offen,2005]



(a)



(b)



(c)

- a similar approach: LCSR for $B \to \pi$ in SCET

  [F.De Fazio, Th. Feldmann and T. Hurth, (2005)]

B-meson DA's

$$\langle 0|\bar{q}_{2\alpha}(x)[x,0]h_{v\beta}(0)|\bar{B}_v\rangle$$
$$= -\frac{if_B m_B}{4}\int\limits_0^\infty d\omega e^{-i\omega v\cdot x}\left[(1+\not v)\left\{\phi_+^B(\omega) - \frac{\phi_+^B(\omega)-\phi_-^B(\omega)}{2v\cdot x}\not x\right\}\gamma_5\right]_{\beta\alpha}$$

- defined in HQET; key input parameter: the inverse moment

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty d\omega \frac{\phi_+^B(\omega,\mu)}{\omega}$$

- QCD sum rules in HQET: $\lambda_B(1\text{ GeV}) = 460 \pm 110$ MeV
  [V.Braun, D.Ivanov, G.Korchemsky,2004]
- all $B \to \pi, K^{(*)}, \rho$ form factors calculated
- so far only tree-level calculations, 2,3-particle DA's

Form factors from LCSR with B-meson DA's

form factor	this work	LCSR with light-meson DA's *[P.Ball and R.Zwicky]*
$f^+_{B\pi}(0)$	0.25±0.05	0.258±0.031
$f^+_{BK}(0)$	0.31±0.04	0.301±0.041±0.008
$f^T_{B\pi}(0)$	0.21±0.04	0.253±0.028
$f^T_{BK}(0)$	0.27±0.04	0.321±0.037±0.009
$V^{B\rho}(0)$	0.32±0.10	0.323±0.029
$V^{BK^*}(0)$	0.39±0.11	0.411±0.033±0.031
$A_1^{B\rho}(0)$	0.24±0.08	0.242±0.024
$A_1^{BK^*}(0)$	0.30±0.08	0.292±0.028±0.023
$A_2^{B\rho}(0)$	0.21±0.09	0.221±0.023
$A_2^{BK^*}(0)$	0.26±0.08	0.259±0.027±0.022
$T_1^{B\rho}(0)$	0.28±0.09	0.267±0.021
$T_1^{BK^*}(0)$	0.33±0.10	0.333±0.028±0.024

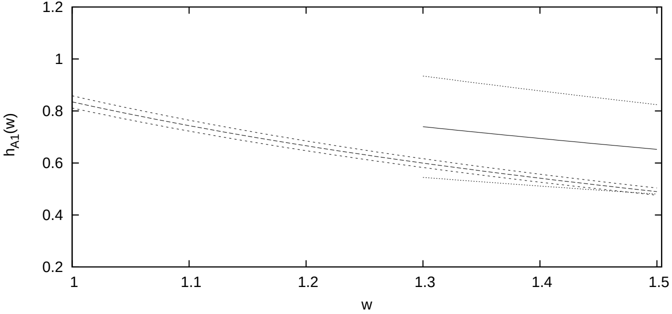
LCSR for $B \to D^{(*)}$ form factors

[S.Faller, A.K.,Ch.Klein, Th.Mannel, paper in preparation]

- virtual c quark in the correlator with B-meson DA
- $B \to D^{(*)}$ form factors near maximal recoil

(not directly accessible in HQET)

- preliminary result for $B \to D^*$ form factor $h_{A1}(w)$



plotted are BaBar data fitted to CLN-parametrization compared to LCSR prediction

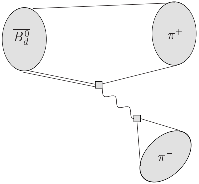
- form factor ratios predicted with a smaller uncertainty

Hadronic Matrix Elements for $B \to h_1 h_2$

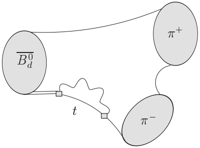
- charmless $B \to h_1 h_2$ channels
(hereafter $B \to \pi K, \pi\pi, ...$):

- a testing ground of quark flavour structure,
interplay of V_{CKM} with complicated hadronic matrix elements,
source of CP-asymmetries
- certain hadronic amplitudes have to be calculated

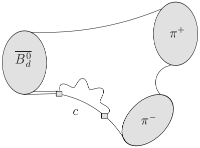
“Anatomy” of $B^0 \to \pi^+\pi^-$



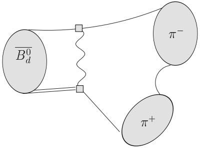
“emission”



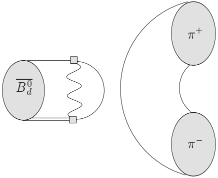
“t-penguin”



“c-penguin”($\oplus$ c$\to$ u)



“annihilation”



“penguin annihilation”

Hadronic amplitudes in $B \to h_1 h_2$

- two-hadron final states not accessible in lattice QCD
- effective theories based on factorization:
  QCDF, PQCD, SCET

(combining $1/m_b$- , α_s- expansions with long-distance inputs)

- a typical QCDF ansatz:

$$\mathcal{A}(\bar{B}_d^0 \to \pi^+\pi^-)_{m_b\to\infty} = i\frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* f_\pi f_{B\pi}(m_\pi^2) m_B^2 \left\{ 1 + O(\alpha_s) + O\left(\frac{\Lambda}{m_b}\right)\right\} ,$$

What can QCD sum rules/ LCSR provide

- the form factors in factorizable amplitudes, e.g., $f_{B\pi}(0)$?
including "soft" contributions

- estimates of charming (u-quark), gluonic penguins and weak annihilation
including $O(1/m_b)$ effects

- estimates of strong phases in $B \to h_1 h_2$
(most difficult and less certain)

Separating short- and long distances

- integrating out electroweak and quark-gluon interactions at scales $\geq \mu \sim m_b$:

$$H_{eff} = \frac{G_F}{\sqrt{2}} \sum_i \lambda_i c_i(\mu) O_i(\mu)$$

c_i -Wilson coeff., $\lambda_i = V_{.b} V^*_{..}$ -CKM ; O_i -eff. operators:
current-current:
$O_1^u = (\bar{d}\Gamma_\mu u)(\bar{u}\Gamma^\mu b), \quad O_2^u = (\bar{u}\Gamma_\mu u)(\bar{d}\Gamma^\mu b),$ also $u \to c$,
$\Gamma_\mu = \gamma_\mu(1-\gamma_5)$
quark-penguins $O_{3,4,5,6}$, EW-penguins $O_{7,8,9,10}$,
magnetic-dipole $O_{7\gamma}$, chromomagnetic-dipole O_{8g}

Decay amplitude

$$A(\bar{B}^0 \to \pi^+\pi^-) = \langle \pi^+\pi^- | H_{eff} | \bar{B}^0 \rangle = \frac{G_F}{\sqrt{2}} \sum_i \lambda_i c_i(\mu) \langle \pi^+\pi^- | O_i | \bar{B}^0 \rangle_\mu$$

- separation of distances: quark-gluon dynamics at
$\langle r \rangle \geq 1/\mu \sim 1/m_b$
contained in hadronic matrix elements of O_i

- Wick contractions of quark fields
$\Rightarrow$ topologies (Emission, Annihilation, Penguin,...),
$\langle \pi^+\pi^- | O_i | \bar{B}^0 \rangle = \sum\limits_{T=E,A,..} \langle \pi^+\pi^- | O_i | \bar{B}^0 \rangle_T$

Light-cone sum rules for $B \to \pi\pi$

*[A. K. Nucl. Phys. B **605** 558 (2001)]*

light-cone OPE $\Leftarrow$ correlator $\Rightarrow$ dispersion relation
$\oplus$ local duality $\Rightarrow$ $\langle \pi^-\pi^+ | O_i | B\rangle_T$

- the correlation function:

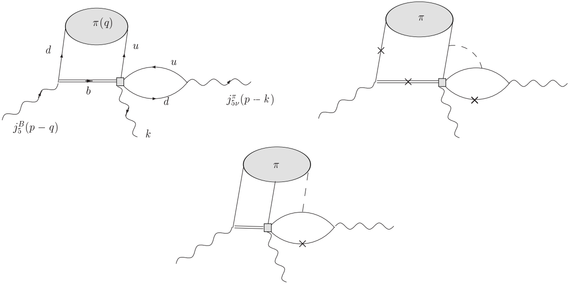
$$F_\alpha^{(O_i)}(p,q,k) = -\int d^4x\, e^{-i(p-q)x} \int d^4y\, e^{i(p-k)y}$$
$$\langle 0 | T\left\{ j^{\pi}_{\alpha 5}(y) O_i(0) j_5^{(B)}(x)\right\} |\pi^-(q)\rangle,$$

O_i-effective operator,
$j^{(\pi)}_{\alpha 5} = \bar{u}\gamma_\alpha\gamma_5 d$ and $j_5^{(B)} = im_b\bar{b}\gamma_5 d$ interpolate π and B

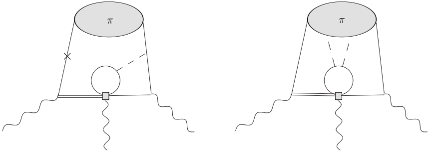
large virtualities $\Rightarrow$ OPE near light-cone $\Rightarrow$ diagrams with pion DA's
the same input as in $B \to \pi$!

OPE diagrams for $O_{1,2}$ operators

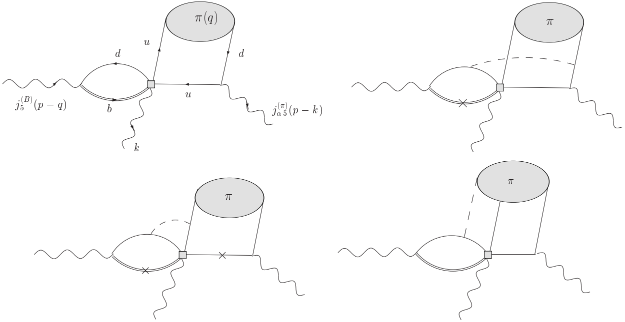
Emission topology



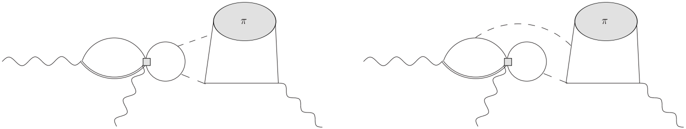
Penguin topology



k-artificial momentum to avoid overlap of B and $\pi\pi$ channels



Penguin Annihilation topology



Hadronic matrix elements for $B \to \pi K, \pi\pi$ from LCSR

- the same input as in LCSR for $B \to \pi, K$:
- size of $O(1/m_b)$ nonfactorizable effects in emission topology
- contributions of penguin and annihilation topologies (with phases!)

$$\mathcal{A}(\bar{B}_d^0 \to \pi^+\pi^-) = i\frac{G_F}{\sqrt{2}} f_\pi \{f_{B\pi}(m_\pi^2)\}_{LCSR} m_B^2$$
$$\times \left\{ \lambda_u(c_1 + c_2/3) + \sum_{k,T} \lambda_k \tilde{c}_k r_{k,T}^{(\pi\pi)} \right\},$$

c_k -short distance coeff, λ_k -CKM ,
$T = E(emission), P(penguin), A(annihilation)$ -topologies,

$$r_{k,T}^{(\pi\pi)} = \frac{\langle \pi^+\pi^- | O_k | \bar{B}^0 \rangle_T}{\langle \pi^+\pi^- | O_1^u | \bar{B}^0 \rangle_E}$$

Results

- (Im part → FSI phase) small
- nonfactorizable emission,

soft gluon- $O(1/m_b)$, hard gluon- $O(\alpha_s)$ (only twist 2)
tw3 hard gluon emission divergent (in simplified version)

$$r_E^{(\pi\pi)} = \left[\left(1.8^{+0.5}_{-0.7}\right) \times 10^{-2} \right]_{soft} + \left[\left(-1.9^{+0.5}_{-0.1} + i \left(-3.6^{+1.0}_{-0.4} \right) \right) \times 10^{-2} \right]_{hard}$$

- charming penguin:

$$r_{P_c}^{(\pi\pi)} = \left[-0.18^{+0.06}_{-0.68} + i \left(-0.80^{+0.17}_{-0.08} \right) \right] \times 10^{-2} \,,$$

- annihilation:

$$r_A^{(\pi\pi)} = \left[-0.67^{+0.47}_{-0.87} + i \left(3.6^{+0.5}_{-1.1} \right) \right] \times 10^{-3}$$

[A. K., T. Mannel, M. Melcher and B. Melic (2005)]

BR's (in units of 10^{-6}) and direct A_{CP}

(very preliminary, not a fit, V_{CKM} from CKM fitter)

	central LCSR prediction	data, HFAG
$BR(B^- \to \pi^- \bar{K}^0)$	15.2	23.1 ± 1.0
$A_{CP}(B^- \to \pi^- \bar{K}^0)$	-0.014	0.009 ± 0.025
$BR(\bar{B}^0 \to \pi^+ K^-)$	12.9	19.4 ± 0.6
$A_{CP}(\bar{B}^0 \to \pi^+ K^-)$	0.07	-0.097 ± 0.01
$BR(\bar{B}^0 \to \pi^0 \bar{K}^0)$	5.5	9.9 ± 0.6
$A_{CP}(\bar{B}^0 \to \pi^0 \bar{K}^0)$	-0.016	-0.14 ± 0.11
$BR(B^- \to \pi^0 K^-)$	8.8	12.9 ± 0.6
$A_{CP}(B^- \to \pi^0 K^-)$	0.06	0.05 ± 0.025
$BR(B^- \to \pi^- \pi^0)$	4.7	5.59 ± 0.41
$A_{CP}(B^- \to \pi^- \pi^0)$	-0.0002	0.06 ± 0.05
$BR(\bar{B}^0 \pi^+ \pi^-)$	7.07	5.16 ± 0.22
$A_{CP}(\bar{B}^0 \pi^+ \pi^-)$	-0.05	0.21 ± 0.09(BaBar)
		0.55 ± 0.08 Belle)
$BR(\bar{B}^0 \to \pi^0 \pi^0)$	0.20	1.31 ± 0.21
$A_{CP}(\bar{B}^0 \pi^0 \pi^0)$	0.77	0.48 ± 0.32

A more refined analysis in progress

- extract the missing (underestimated) hard scattering term from the ratio of $B^- \to \pi^-\pi^0$ vs semileptonic BR
- fit the data with CKM fitter
- add an addiitonal phase between $I = 1/2$ and $I = 3/2$

(work in progress with M.Jung and B.Melic)

QCD sum rules: a summary

- suggested almost 30 years ago

  [M.Shifman, A.Vainshtein and V.Zakharov (1979)]

a method to parametrize QCD vacuum effects,
e.g., $\sim$ 90% of $m_{nucleon}$ is due to quark condensate

- hadrons treated indirectly

  (poles in the correlation function)

- initially not intended for precision calculations
- nowadays a popular working tool:

many applications, new modified approach (LCSR)

- applications to flavour physics demand

high accuracy

- uncertainties have to be identified and estimated

How accurate are QCD sum rules

- two main sources of uncertainties:

  (I) OPE: truncated, inputs uncertain

- a reasonable accuracy achieved in 2-point correlators,
due to progress in multiloop calculations,

- α_s, quark masses, quark/gluon condensates, DA's:
accuracy slowly improving

- in LCSR only NLO $t \leq 4$ available,
twist expansion demands additional studies

(II) hadronic sum approximated with quark-hadron duality

- more difficult,

the most safe predictions are bounds from OPE

- not easy to estimate the "systematic" error

related with effective threshold s_0:
(fitting s_0 by adjusting the hadron mass)

- a better solution: experimental information

on excited states $\Rightarrow$ the hadronic spectral function

- "Borel window":

M^2_{min} (subleading power terms small)
M^2_{max} (suppression of excited states)
sometimes does not exist !

QCD sum rules beyond QCD ?

- hypothethical new strong interaction sector beyond SM (e.g. technicolor-like)
- if theory is asymptotically free and predicts the existence of new hadronlike bound states,
- sum rules can be used to fix the properties of those states by matching the correlators with hadronic sum