A New Approach towards Detecting Community Structures in Networks
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A report based on joint work with

William Y.C. Chen and Winking Q. Yu

from the Center for Combinatorics, LPMC, Nankai University, Tianjin 300071, P.R. China.
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Dynamical Systems and Networks

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Networks are *snapshots* of dynamical systems and dynamical systems are networks *in action*.

Consequently, there is some good hope that proper network analysis can help to elucidate a system’s dynamics.
Standard Approaches in Network Analysis

Standard methods of network analysis require a lot of detailed input information about the mechanisms of interactions between the various agents participating in the network's activity as well as the respective inter-and reaction rates. Given such information, a lot of detailed information about the dynamics of actual processes can then be deduced by solving the resulting (ordinary and/or partial differential) equations or by mimicking the interaction schemes by computer simulation.
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So, what can be done if all that is (more or less) known are — the network’s agents represented by a collection $V$ of nodes,

— and the network’s topology, i.e., the subset $E$ of the set $\binom{V}{2}$ of all 2-subsets $\{u, v\}$ of $V$ consisting of those pairs of distinct agents $u, v$ that we believe to be closely related to, or to strongly interact with, each other (also called the edges of the network)?
Zachary’s Karate Club from 1977

partition line of the communities
Community-Structure Detection

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or, alternatively phrased, for grouping the network’s agents into disjoint communities consisting of agents that appear to strongly interact with each other, and not so strongly with the agents in the other communities,

with the aim of, e.g., predicting this partition line from the topology of the “friendship” network.
The Current Network Hype

Methods for detecting community structures in networks have received much attention ever since the current network hype began with the proclamation of \textit{scale-free} and \textit{small-world} networks.
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The Current Network Hype II

According to SCIENCE CITATION INDEX EXPANDED, D.Watts and S.Strogatz’ paper on “Collective dynamics of ’small-world’ networks” and A.Barabasi and R.Albert’s paper on “Emergence of scaling in random networks” both have been quoted more than 1500 times.
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Here is once again the example of Zachary’s Karate Club
Zachary’s Famous Karate-Club Example Published in 1977
Recent Approaches

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**Community structure in social and biological networks**
Recent Approaches II

resulting in a flurry of many further papers by, e.g.,

- J.R. Tyler, D.M. Wilkinson, and B.A. Huberman,
- F. Wu and B.A. Huberman,
- F. Radicchi, C. Castellano, F. Cecconi, V. Loreto, and D. Parisi

and many more.

Then, M. Newman and M. Girvan introduced the modularity parameter to quantify how well a given community structure fits a given network that led to

- a first modularity-optimization algorithm
- that was later improved by the ”CNM-algorithm” developed by A. Clauset, M. Newman, and C. Moore and tries to find community structures of high modularity by greedy optimization.
- And more work was done by J. Reichardt and S. Bornholdt, J.P. Bagrow and E.M. Bollt, and A. Clauset again who developed a method for finding “local” community structures.
A Strategy from 1989

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For short, any such network will henceforth be dubbed a target network.

Clearly, there is a canonical one-to-one correspondence between target networks with node set $V$ on the one, and set partitions of $V$ on the other hand.
The Indicator Function of Target Networks

In consequence, using a standard book-keeping device and describing the edges of a graph $G = (V, E)$ with node set $V$ and edge set $E \subseteq \binom{V}{2}$ in terms of the associated indicator function

$$\chi_G : \binom{V}{2} \rightarrow \{0, 1\} : \{u, v\} \mapsto \chi_G(uv) := \begin{cases} 1 & \text{if } \{u, v\} \in E, \\ 0 & \text{else,} \end{cases}$$

a network $T = (V, F)$ is easily seen to be a target network if and only if the linear inequality

$$\chi_T(uv) + \chi_T(vw) - \chi_T(uw) \leq 1$$

is satisfied, for any three distinct nodes $u, v, w \in V$, by its indicator function $\chi_T$. 
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is satisfied, for any three distinct nodes $u, v, w \in V$, by its indicator function $\chi_T$. 
In other words, using the indicator function allows us to simply reformulate a geometric-combinatorial fact in purely algebraic-numerical terms.
How to Append and to Eliminate Edges in a Most Parsimonious Way to Obtain a Target Network?

Consequently, all that still needs to be done is to find out how, given a network $G = (V, E)$ as above, we can insert and eliminate edges in a *most parsimonious* way so that the resulting network becomes a target network.
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The most simple way to measure the deviation of the original network $G = (V, E)$ from a given target network $T = (V, F)$ is, of course, the total number of switched (i.e., of inserted or eliminated) edges, a number that is easily seen to coincide with the term
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$$\sum_{\{u,v\} \in E} (1 - \chi_T(uv)) + \sum_{\{u,v\} \in \binom{V}{2} - E} \chi_T(uv)$$
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giving rise to a penalty function that is apparently an affine linear function of the indicator function $\chi_T$. 
The Resulting Integer Linear Programming Problem

So, following the approach worked out so excellently by M. Grötschel and Y. Wakabayashi, we can use integer linear programming (ILP) to find (the indicator function of) an optimal target network relative to that penalty function.
Variations

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We are allowed to specify, for every 2-subset \( \{u, v\} \in \binom{V}{2} \) of \( V \), an arbitrary positive or negative number \( L_{a \text{ priori}}(uv) \) registering an \textbf{a priori} measure for the likelihood of the pair \( u, v \) being contained in the same community within the community structure we want to detect,
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\[
L(T) := \sum_{\{u,v\} \in \binom{V}{2}} \chi_T(uv)L_{\text{a priori}}(uv).
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Variations II

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In particular, it may be tempting to experiment with the various edge parameters used in the work by M.Newman and others referred to above.
Our Current ’Ansatz’

Currently, we are using the “CPLEX” software package to investigate this approach,

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Here, deg_G(x) is of course, for any node x in a graph G = (V, E), the number of edges that are incident with it, and s is a positive real number that we use for appropriately calibrating our objective function.
Currently, we are using the “CPLEX” software package to investigate this approach, experimenting, just for a start, with a parameterized \textit{a priori} likelihood function of the form

\[ L_{\text{a priori}}(uv) := \begin{cases} 
-s \left( \deg_G(u) + \deg_G(v) \right) & \text{if } \{u, v\} \in E, \\
2(|V| - 1) - \deg_G(u) - \deg_G(v) & \text{else.}
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An Unexpected 'Phase Transition'

Remarkably, increasing the control parameter $s$ from 1 to larger and larger values, the running time of the ILP problem becomes shorter and shorter until a value, say, $s^*$ is found for which
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- the running time of the associated ILP problem is approximately that of the corresponding relaxed LP problem,
- the solutions of both problems coincide (i.e., the relaxed problem has an integral solution),
- and the community structure resulting in case $s = s^*$ has, so far, consistently turned out to basically coincide with that community structure which is considered to be the "correct" one by other researchers.
An Unexpected 'Phase Transition' II

To give an example, let us go back to the well known data regarding “Zachary’s Karate Club” that is specified in terms of a simple graph with 34 nodes:
The Famous Karate-Club Example

partition line of the communities
The (final) result of our algorithm reproduces exactly the same partition line that represents the real-world situation.
Results

The (final) result of our algorithm reproduces exactly the same partition line that represents the real-world situation.

And it results at $s^* := 38.8$.

The next diagrams show, as functions of $s$,

- the time CPLEX needs to find a solution,
- the proportion between the optimal value of the ILP problem and the associated “relaxed” LP problem,
- and that between the respective computation times.
The Computation Time as a Function of $s$
The ILP/LP Objective Function Ratio as a Function of $s$
The ILP/LP Computation Time Ratio as a Function of $s$
Improving the Speed of our Algorithm

In consequence, we propose now to use the following variant of our algorithm:

\[ s := 1, \text{ run the relaxed LP program for larger and larger values of } s \text{ until, for the first time, you'll find an integer solution.} \]

And accept the associated target graph as the community structure you search for.
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The Chesapeake Bay Food Web

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We also compare, in the next slide, our result with that of the GN-algorithm.
The Network and the (Principal) Result of our Algorithm
A Comparison with the GN-Algorithm

Benthic Organisms

Pelagic Organisms

Undetermined

four errors

six errors

four undetermined
Oct. 24, 2006

Dear Andreas and Winking,
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I finally had the chance to look at the groupings that you sent to me. To begin with, I noticed only a single transposition difference between your grouping and that by Girvan and Newman. Namely, you group blue crab (19) in the second group and blue fish (30) in the first. GN does the opposite.
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My judgement is that both groupings are quite good, but yours probably wins out by a hair.
Please notice that the groupings are according to where the organisms “feed”, not where they are located. Hence, three of the ”mistakes” you indicated are not mistakes at all by my reckoning.
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That mya (12), oysters(13) and other suspension feeders (11) live on the bottom is only incidental. They are all filter feeders and take their nourishment from the water column. In terms of feeding, they belong with the pelagic organisms.
A Letter from Robert Ulanowicz II

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On the other hand, spot (27), white perch (28), croaker (25), catfish (29) and hogchoker (26) are technically nektonic, but they all derive their nourishment primarily from the benthos and can logically be placed in the second category.
As for the single discrepancy between the two methods, I would judge that blue crabs (19) belong decidedly in the benthic feeding group, as your method detected.
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Blue fish (30) feed mostly on other nekton, but ultimately derive most of their sustenance from the benthos.

In fact, in the paper I sent you on Oct. 10, we note how the indirect diet of striped bass (33) differs from that of blue fish (30) because the former derives most of its sustenance from the pelagic domain, whereas the latter comes ultimately (but not directly) from the benthos.
Hence, blue fish is a "borderline" species, and GN do not err gravely by placing it among the benthic feeders. Their bigger error is in placing blue crab (19) among the pelagic feeders.
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So by placing blue crab correctly among the benthic feeders, you win by a slight edge. :)

I do hope these observations have been helpful.

I am impressed by the power of your grouping algorithm.

Sincerely, Bob
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A Perturbation Experiment

Finally, some very recent results that we obtained studying the "reconstructibility" of a given target graph $T = (V, E)$ from graphs $G$ obtained from $T$ by systematically perturbing $T$:

We choose the disjoint union of 4 cliques containing 12, 9, 8, 6 nodes, respectively, — and, hence, altogether $66 + 36 + 28 + 15 = 145$ edges — as our target graph $T$. 
A Perturbation Experiment II

To explore how well this graph is reconstructed from the perturbed graphs $G$, we
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To explore how well this graph is reconstructed from the perturbed graphs $G$, we

(1) considered randomly generated graphs $H = (V, F)$ whose edge sets were, in each instance, supposed to represent all the edges that were to be switched, and formed the graph

$$G := (V, (E - F) \cup (F - E)),$$

(2) applied our algorithm to $G$ yielding a target graph $T' = (V, E')$,

(3) and considered the resulting maintenance ratio relative to $H$, i.e.,

the quotient of (i) the (minimal) number of nodes that have to be moved from one clique to another one to obtain $T$ from $T'$ and (ii) the number $|V|$ of all nodes in $V$ (which happens to be 35).

The following table lists the average maintenance ratio obtained for (ten) random graphs $H = (V, F)$ for any given number $|F|$ of edges or, equivalently, for any given perturbation ratio, that is, the quotient $|F|/|E| = |F|/145$, using in addition the most simple possible objective function defined by putting

$L_{a priori}(uv) := \begin{cases} 1 & \text{if } \{u, v\} \in E, \\ -s & \text{else}. \end{cases}$
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$$
L_{\text{a priori}}(uv) := \begin{cases} 
1 & \text{if } \{u, v\} \notin E, \\
-s & \text{else}.
\end{cases}
$$
The perturbation and the maintenance ratio

<table>
<thead>
<tr>
<th>Number of switched edges</th>
<th>Perturbation ratio</th>
<th>Maintenance ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.103</td>
<td>1</td>
</tr>
<tr>
<td>29</td>
<td>0.200</td>
<td>1</td>
</tr>
<tr>
<td>44</td>
<td>0.303</td>
<td>1</td>
</tr>
<tr>
<td>51</td>
<td>0.352</td>
<td>1</td>
</tr>
<tr>
<td>53</td>
<td>0.366</td>
<td>1</td>
</tr>
<tr>
<td>55</td>
<td>0.379</td>
<td>1</td>
</tr>
<tr>
<td>58</td>
<td>0.400</td>
<td>1</td>
</tr>
<tr>
<td>73</td>
<td>0.503</td>
<td>1</td>
</tr>
<tr>
<td>87</td>
<td>0.600</td>
<td>0.99</td>
</tr>
<tr>
<td>102</td>
<td>0.703</td>
<td>1</td>
</tr>
<tr>
<td>116</td>
<td>0.800</td>
<td>0.98</td>
</tr>
<tr>
<td>131</td>
<td>0.903</td>
<td>0.63</td>
</tr>
<tr>
<td>145</td>
<td>1</td>
<td>0.36</td>
</tr>
</tbody>
</table>
A Comparison with the CNM Algorithm

![Graph showing a comparison between the LP method and the CNM method.]

- **Perturbation Ratio**
- **Average Maintenance Ratio**
- **LP method**
- **CNM method**

The graph compares the performance of the LP method and the CNM method across different perturbation ratios. The CNM method shows a more stable performance compared to the LP method, especially as the perturbation ratio increases.
Dear Andreas,

If your work is going to be presented in the computer science community, it would be most correct to call our approach a heuristic; the physics community is less picky, so there, it would be an algorithm.

You could also note that it is not an optimal greedy algorithm, and that it is known to return poor results for certain pathological networks that appear to be unlike those we see in the real world. It doesn't really matter how big (or small) the pathological network is, the greedy approach will always give you a bad answer.
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In either case, it would be most correct to say that it is a very fast greedy approach to the maximum-modularity problem, and that it returns demonstrably good results for many real-world networks.
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In either case, it would be most correct to say that it is a very fast greedy approach to the maximum-modularity problem, and that it returns demonstrably good results for many real-world networks.

You could also note that it is not an optimal greedy algorithm, and that it is known to return poor results for certain pathological networks that appear to be unlike those we see in the real world. It doesn’t really matter how big (or small) the pathological network is, the greedy approach will always give you a bad answer.
So, the most accurate characterization would be to say that, as the size of the network you want to cluster increases, you have progressively fewer choices of algorithms for maximizing the modularity (or any other of those nice parameters used for community-structure detection) because most of them take time at least $O(n^2)$, and we want our algorithms to return an answer after a reasonable amount of time.
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For the very largest networks, say on the order of hundreds of thousands or millions of nodes, basically only our greedy algorithm will return an answer to you within this constraint.
Directions for Future Work

Our results imply various questions that deserve to be investigated further and suggest several tasks that deserve to be pursued in the future:

▶ Optimizing the use of the CPLEX program,
▶ studying a larger range of objective functions and trying to determine those that seem to be particularly appropriate for a specific task, including objective functions related to edge-weighted networks and asymmetric ones representing directed networks,
▶ trying to understand the influence exercised by, and in particular the apparent “phase transition” behaviour of, the control parameter $s$,
▶ analysing the “landscape” defined on the set of target graphs by a given ($s$-parametrized) objective function using stochastic models and, in particular, the entropy concept from statistical physics,
▶ trying to understand also the apparent “phase transition” behaviour of the maintenance ratio relative to the perturbation ratio,
▶ developing approximative algorithms for large-scale applications,
▶ creating a data base containing the results obtained by applying the algorithm(s) to real-world data gathered from the existing literature.
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Thanks

Thank You for Your Patience and Attention!
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Xie, Xie ! !! !!!
References


