Quantum phases of flexible quasi-molecular strings of ultracold atoms

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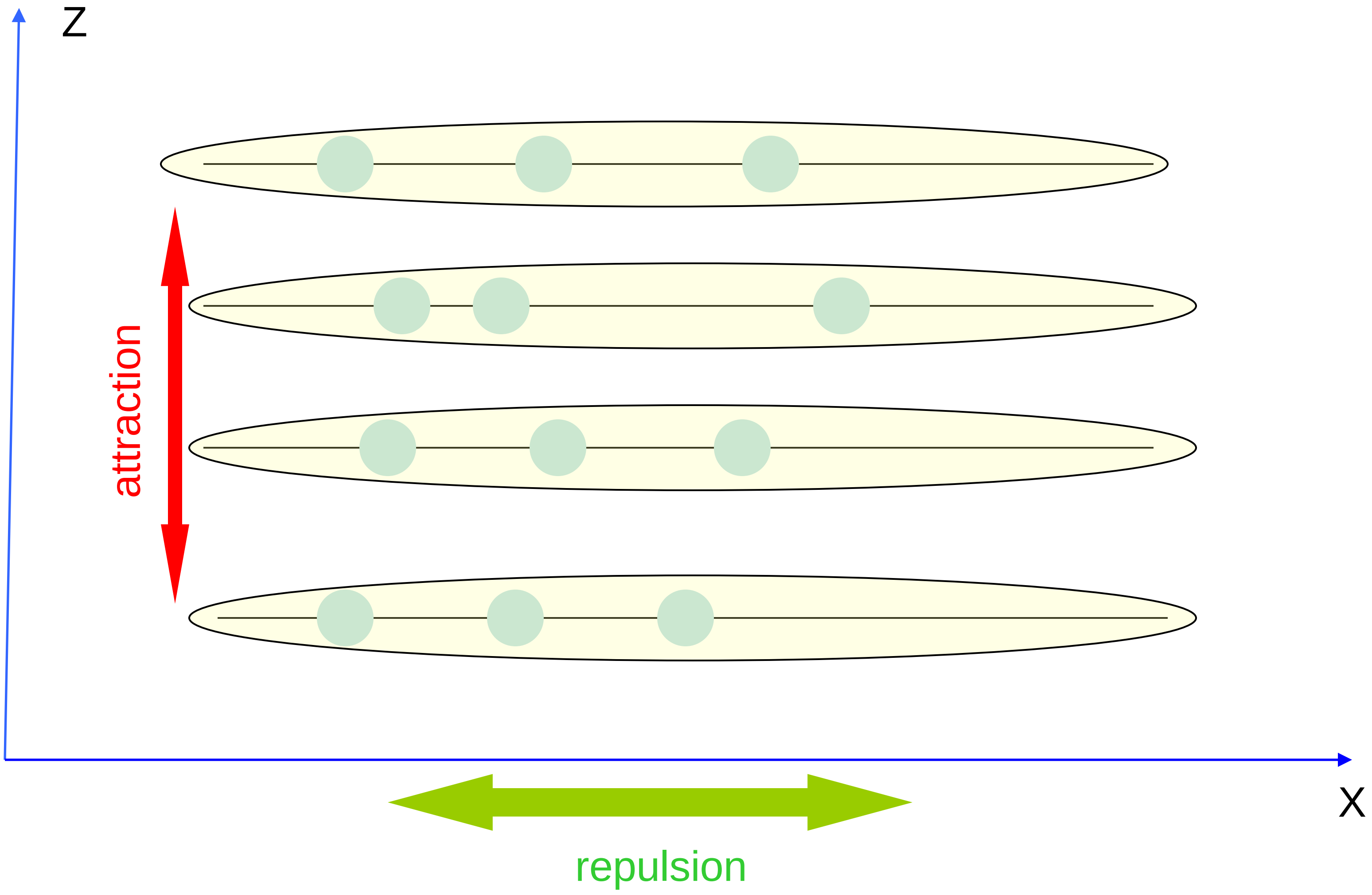
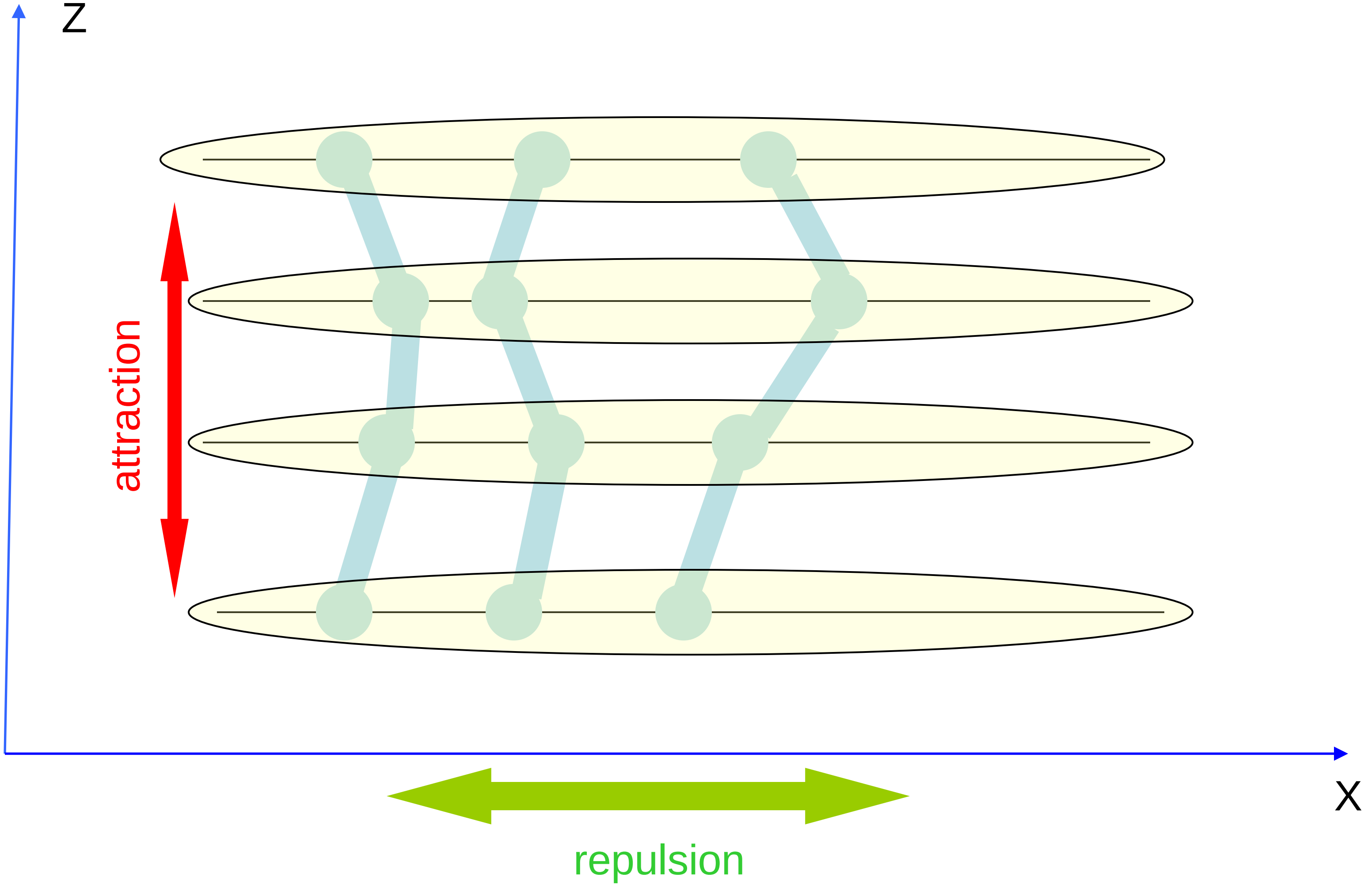
Dipolar interactions = Attraction along Z and repulsion along X:

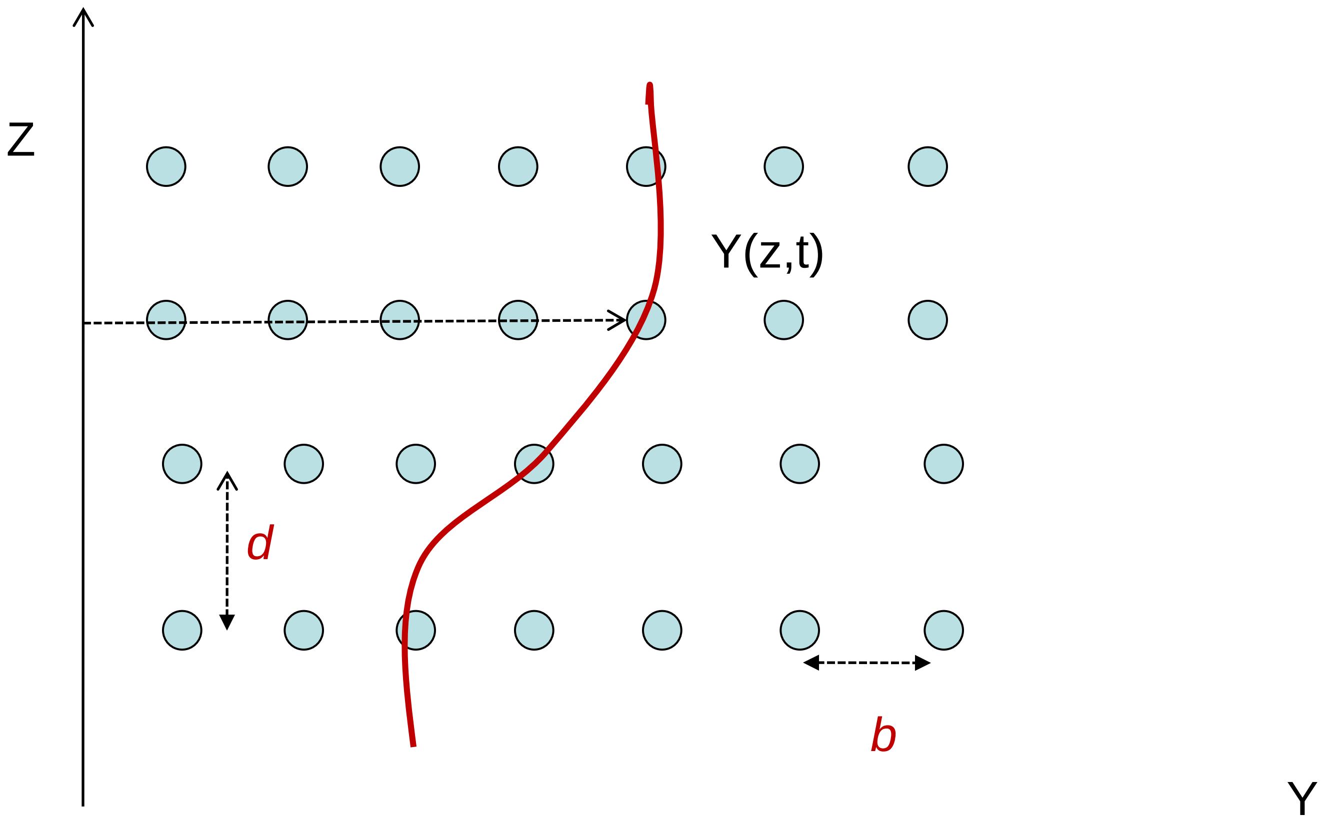


Flexible quantum chains



Rigid chains: **D.-Wei Wang, et al PRL. 97, 180413 (2006)**

Single chain in OL at T=0



$$\frac{\hbar}{mb^2} \geq \frac{V_s}{d}$$

: Lattice is irrelevant and the chain is free

Berezinskii-Kosterlitz-Thouless transition at T=0

Two free chains at T=0

$$L = \int dz \left\{ \frac{K}{2} \left[\partial_t \theta^{\ 2} - \partial_z \theta^{\ 2} \right] - V(\theta) \right\}$$

$$\theta \sim Y_1 - Y_2$$

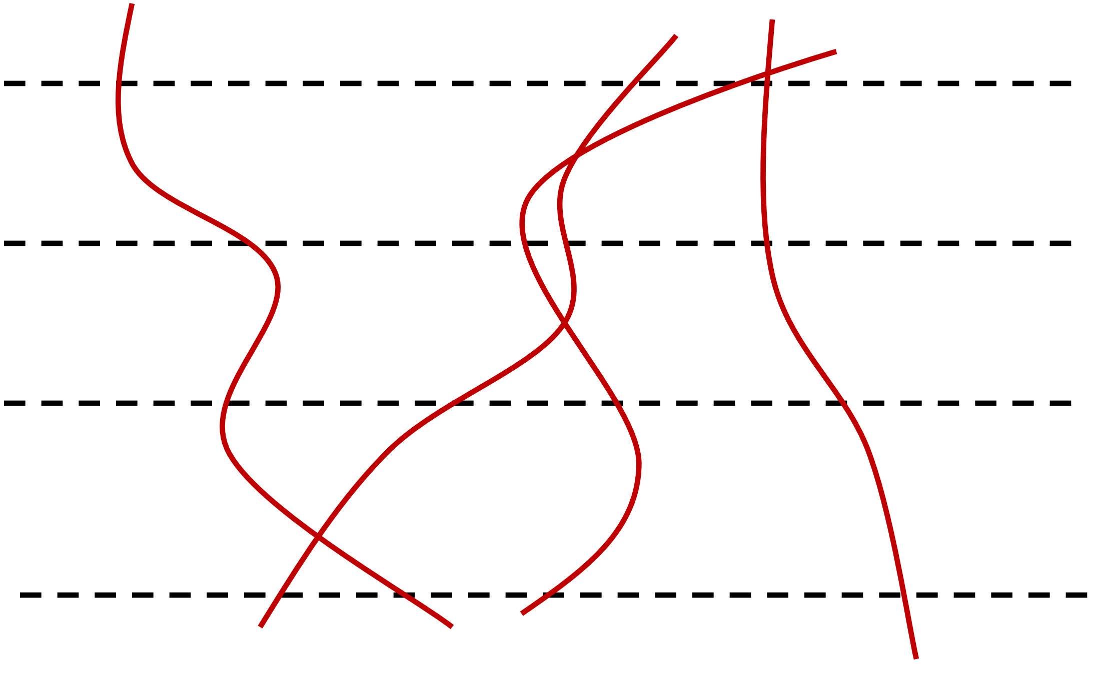
$V(\theta) -$ non-periodic attractive potential

$$V(\theta) \sim -u_o \exp(-\lambda \theta^2)$$

Two attractive chains are always bound to each other
for arbitrary small interaction u_o

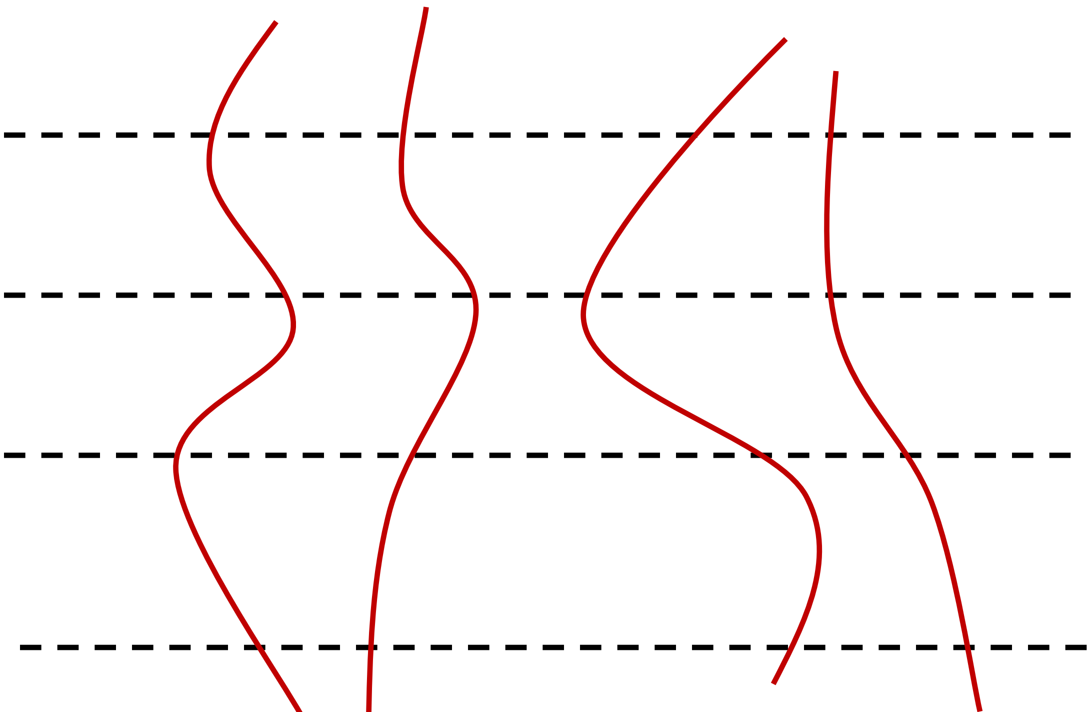
States of repulsive chains at T=0 ???

Superfluid



States of repulsive chains at T=0 ???

Ordered insulator



Hamiltonian

$$H = -t \sum_{<ij>,b} a^+_{ib} a_{jb} - U \sum_{<bb'>,i} n_{ib} n_{ib'} + V \sum_{b,i} n_{ib} n_{ib}$$

$$n_{ib} = a^+_{ib} a_{ib}$$

i - intra-layer position
b – layer label

Order parametrs

$$\text{N-component SF}: \langle a_{i,b} \rangle \neq 0, \quad b = 1, 2, \ldots N$$

$$\text{M-atomic molecule:} \quad \langle a_{i,1} a_{i,2} a_{i,3} \ldots . a_{i,M} \rangle \neq 0, \quad M = 2, 3, ..??$$

D=1+1 Bosonization

A. Vishwanath, D. Carpentier, PRL **86**, 676(2001).
L. Mathey, et al, PRA **79**, 011602R 2009

$$\psi_b(x) = (n + \Pi_b(x))^{1/2} \sum_{m=0,1,2...} \exp(2mi\theta_b(x)) \exp(i\varphi_b(x))$$

$$S = \iint dxdt \sum_{b=1,2,...N} \left(\frac{1}{2\pi K} \ (\partial_t \theta_b \ \ ^2 + (\partial_x \theta_b)^2) + g' \ \partial_x \ \ \theta_b - \theta_{b+1} \ \ ^2 - g\cos(2\theta_b - 2\theta_{b+1}) \right)$$

$$K < K_c = \frac{1}{2} \sum_{n=1,2,..N-1} \sin^2\left(\frac{\pi}{N} n\right)$$

$$S = \iint dxdt \sum_{b=1,2,...N} \left(\frac{1}{2\pi K} \ (\partial_t \theta_b \ \ ^2 + (\partial_x \theta_b)^2) + g' \ \partial_x \ \ \theta_b - \theta_{b+1} \ \ ^2 + 2g(\theta_b - \theta_{b+1})^2 \right)$$

$$\tilde{\theta}_{q_z}(x,t) = \frac{1}{\sqrt{N}} \sum_{b=0,1,2...N-1} e^{iq_z b}\theta_b(x,t)$$

$$\Delta(q_z) \sim \sqrt{g\sin^2\ q_z/2}\ , \quad q_z = \frac{\pi}{N}n, \quad n = 0,1,2,...N-1$$

$q_z = 0,$ N-atomic molecular (chain) condensate!!!

$$\langle \psi_{\mathrm{b}} \rangle = 0, \quad \langle \psi_1 \psi_2 ....\psi_N \rangle \neq 0$$

D=2+1: J-current mapping

$$H_J = \sum_{i,b} \mathrm{V}\, \vec{J}_{i,b}^{\,2} - U \sum_{<bb'>,i} \vec{J}_{i,b} \vec{J}_{i,b'} - \mu \sum_{i,b} J_{i,b}^{(t)}$$

$$\vec{J}_{i,b} = (J_{i,b}^{(x)}, J_{i,b}^{(t)}); \qquad \vec{\nabla}\vec{J} = 0, \qquad |J| = 0, 1, 2, \ldots$$

N-component SF: $\left\langle \vec{W}_b^{\,2} \right\rangle \neq 0, \ \left\langle \vec{W}_b \vec{W}_{b'} \right\rangle = 0, \quad b = 1, 2, \ldots N$

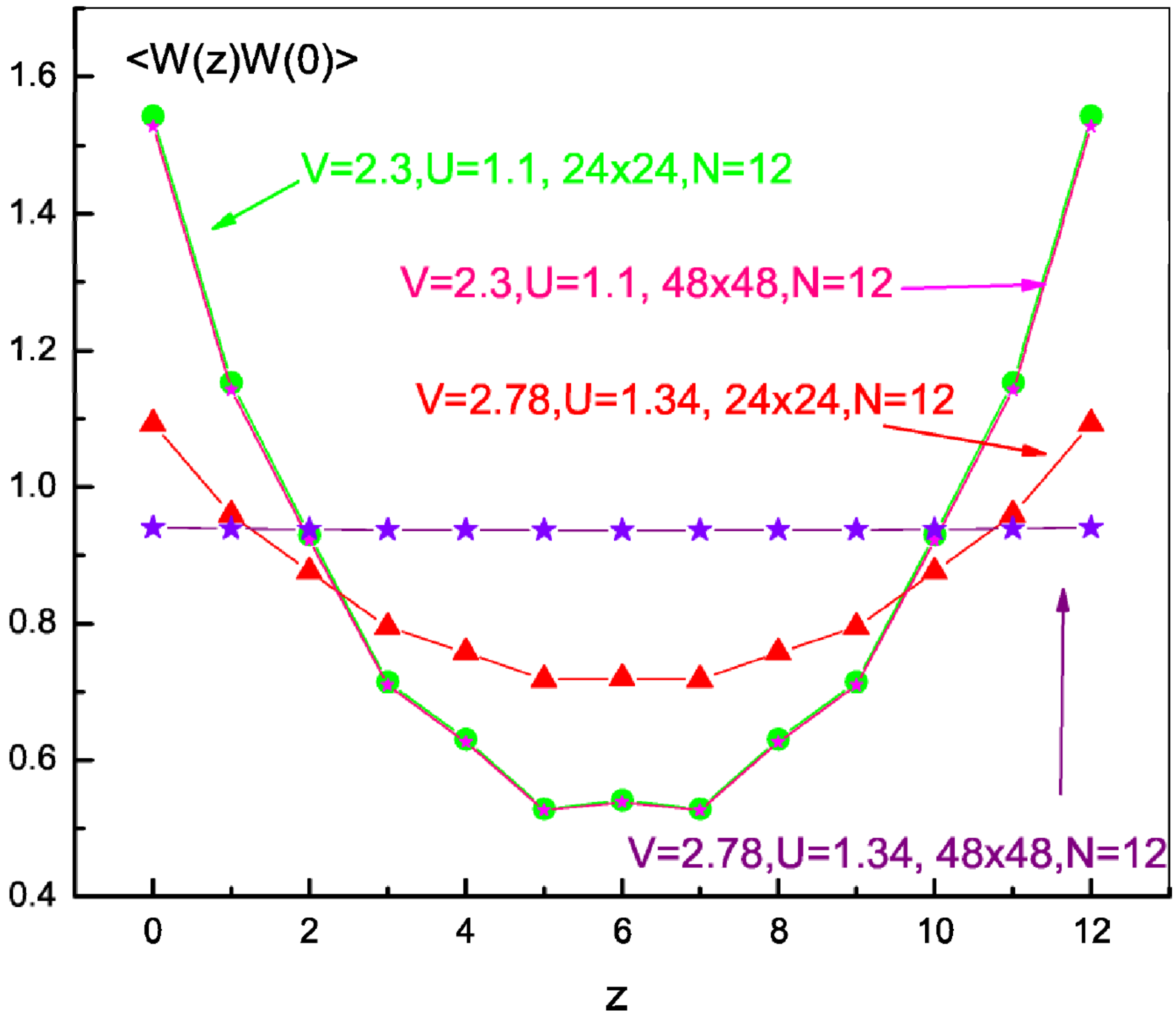
$\vec{W}_b$ – winding numbers = total current

Quantum chains SF:

$$\vec{W}_q = \sum_{b=1,..N} e^{ibq} \vec{W}(b), \quad q = \frac{2\pi}{N} n, \qquad n = 0, 1, 2, \ldots N-1$$

$$\vec{W}_{q=0} \neq 0, \quad \vec{W}_{q\neq 0} = 0$$

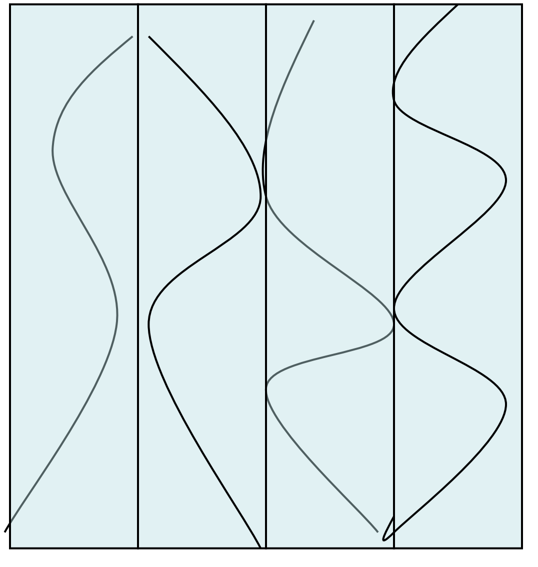
Quantum phase transition into the chain SF in D=1+1 and D=2+1 by Worm Algorithm



What happens at non-integer filling?

Is it always the chain SF at T=0?

Caging effect: shape fluctuations induce non-local repulsion



Backscattering in D=1+1 (F.D.Haldane, Phys. Rev. Lett. **47**, 1840 (1981)):

$$S_{back} \sim \sum_{b,x} \cos(2\theta_b + 2\pi n_b x)$$

For non-integer filling n_b<1 *always D=1+1 SF*

For $n_b=1$ – Mott transition at K<1/2

Backscattering for strings in D=1+1:

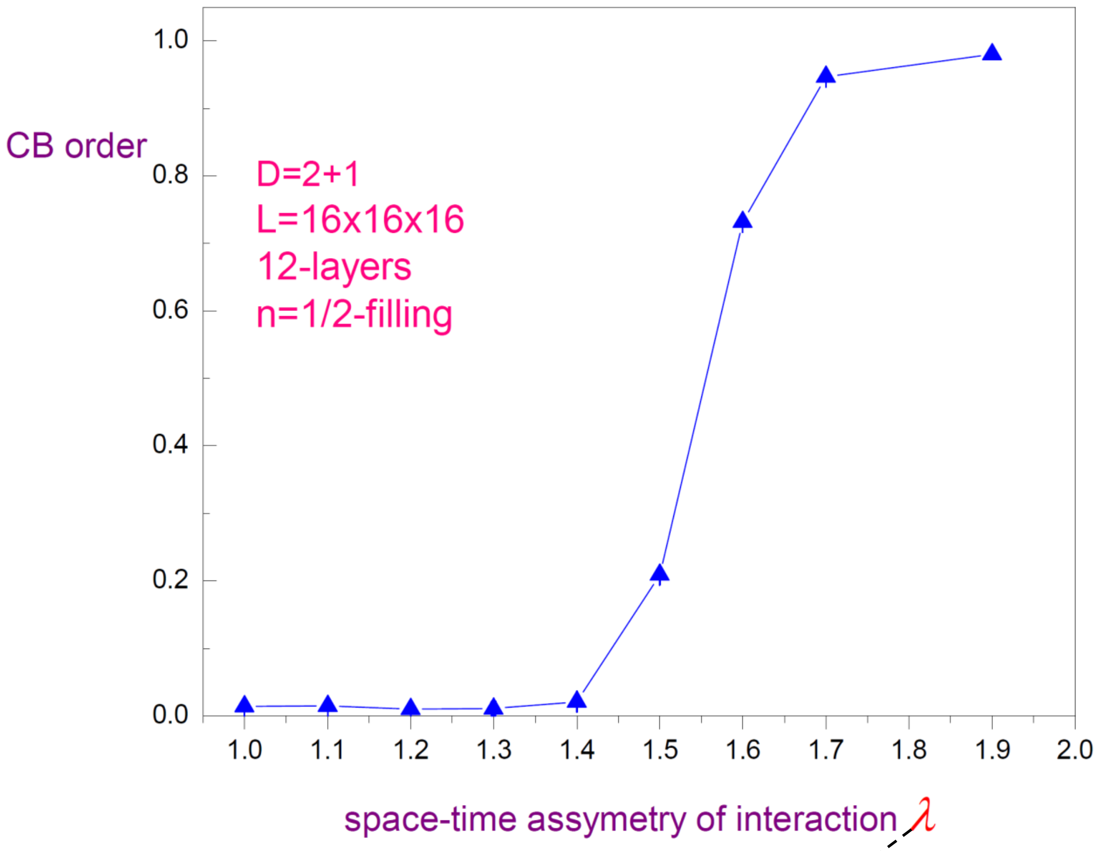
$$\Phi_{str}(x) = \prod_{b=1,2,..N} \psi_b(x) \sim \exp(i2\sum_{b=1,..,N}\theta_b)\exp(i\sum_{b=1,...N}\phi_b) = \exp(2i\theta_{str})\exp(i\phi_{str})$$

$$S_{bksc} \sim \int d\tau \sum_x \cos(2\theta_{str})$$

$$\theta_b = \pi n x + \Theta_b \rightarrow \cos(2\Theta_{str} + 2\pi n N x)$$

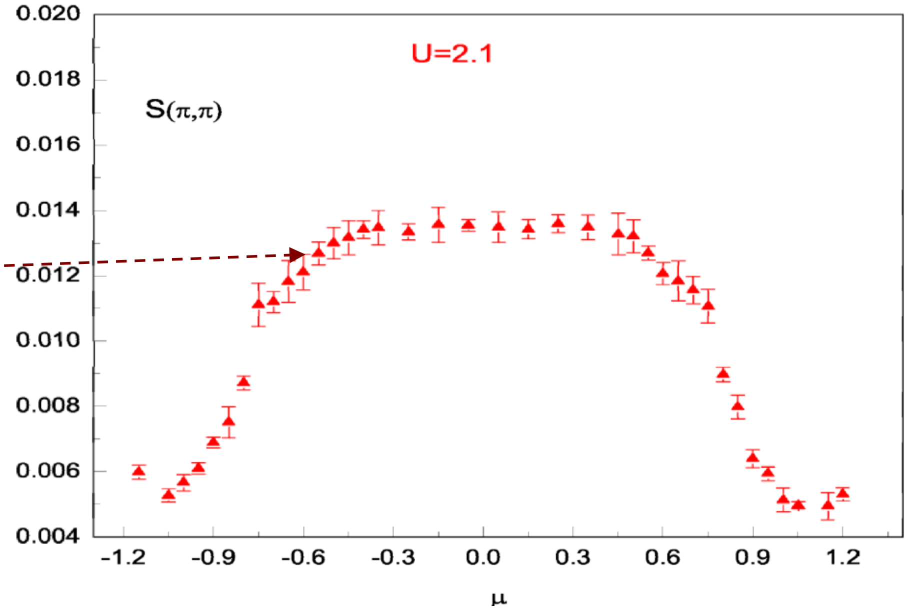
For *nN*=1,2,3,... external periodic potential can localize strings!!!

Transition: Checkerboard Insulator in D=2+1-*quantum disordered state??!!*

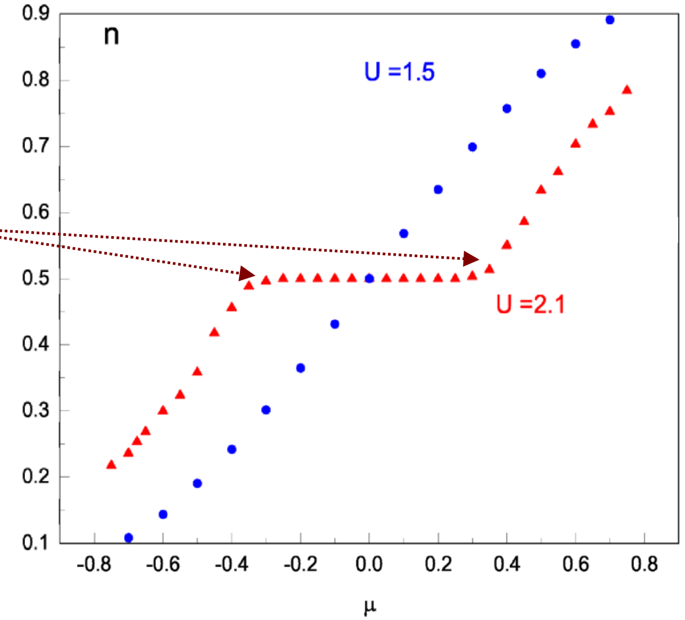


$$H_J = \sum_{i,b} V\, \vec{J}^{\,2}_{i,b} - U \sum_{<bb'>,i,space} \vec{J}_{i,b}\vec{J}_{i,b'} - U\lambda \sum_{<bb'>,i} J^{(\tau)}{}_{i,b} J^{(\tau)}{}_{i,b'} - \mu \sum_{i,b} J^{(t)}_{i,b}$$

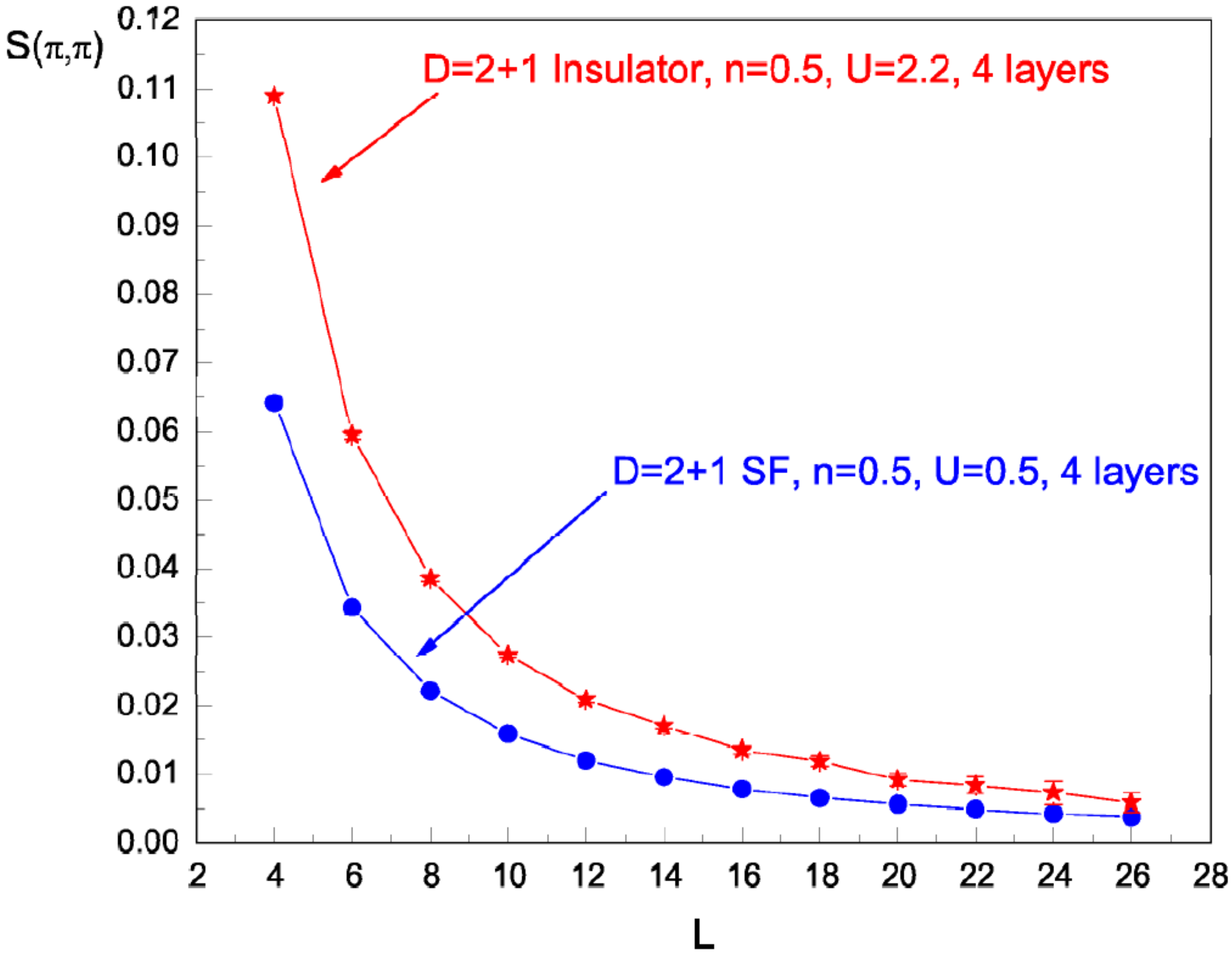
Very weak checkerboard order decreasing with size ~ 1/L



Gap (10 layers)



Absence of checkerboard order in thermodynamical limit



Strong inter-layer attraction :

$$U > 2.0$$

Transition to a phase where
compressibility and superfluid stiffness=0 at filling <1

Q: What is this phase?

Incompressible states without any order at non-integer fillings!!!!????

Summary

- Dipolar forces and flexible quantum strings
- Superfluidity of flexible strings
- Incompressible (insulating) state without any obvious diagonal order at incommensurate filling for N>2
- Cage effect