

# Induced interaction in a Fermi gas with a BEC-BCS crossover

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(Z.-Q. Yu, K. Huang, and L. Yin, PRA **79**, 053636 (2009))

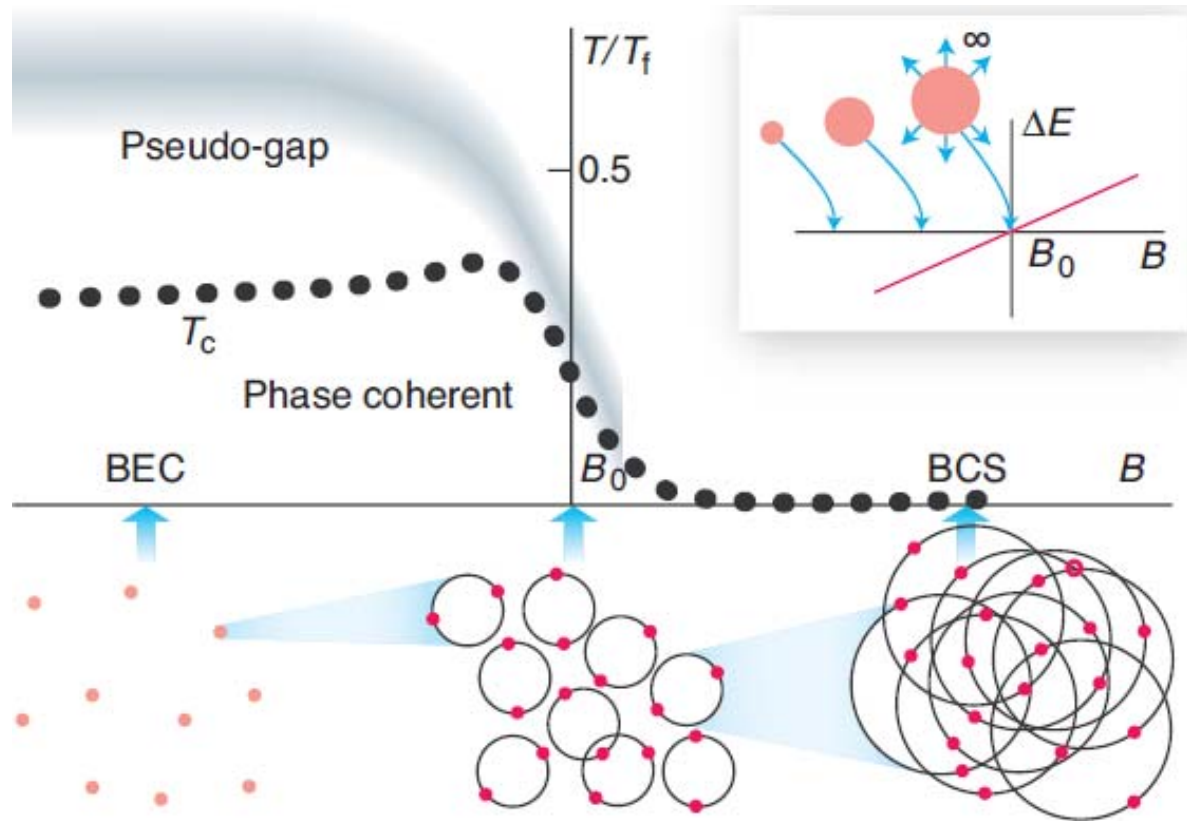
# Outline

- (1) Introduction
- (2) Induced interaction for BEC-BCS crossover
- (3) Effect on superfluid transition temperature  $T_c$
- (4) Conclusion

# (1) Introduction

## BEC-BCS crossover

(JILA, MIT, Innsbruck, Duke, Rice...)



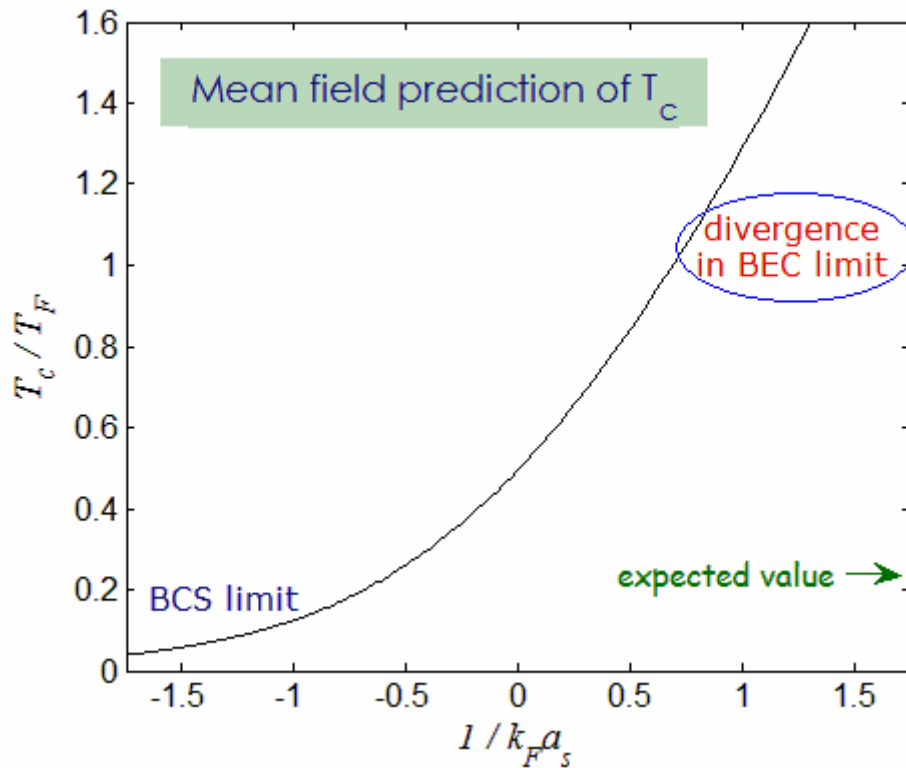
T.-L. Ho, Science 305, 1114 (2004)

# Mean-field $T_c$ of BEC-BCS Crossover

## Single-channel model

$$H = -\sum_{\sigma} \psi_{\sigma}^{\dagger} \frac{\hbar^2 \nabla^2}{2m} \psi_{\sigma} + g \psi_{\uparrow}^{\dagger} \psi_{\downarrow}^{\dagger} \psi_{\downarrow} \psi_{\uparrow}, \quad g = 4\pi \frac{\hbar^2}{m} a_s$$

**Tc-Equation**  $\frac{1}{g} = -\int \frac{d^3k}{(2\pi)^3} \left[ \frac{\tanh(\beta \varepsilon_k / 2)}{2\varepsilon_k} - \frac{1}{2(\varepsilon_k + \mu)} \right], \quad \varepsilon_k = \frac{\hbar^2 k^2}{2m} - \mu$



BCS limit

$$T_C \approx 0.61 T_F \exp[\pi / (2k_F a_s)]$$

BEC limit

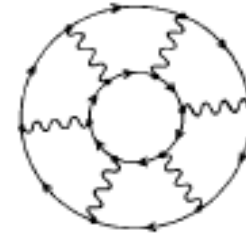
$$k_B T_C \approx -\hbar^2 / [2ma_s^2 \ln(na_s^3)] \gg E_F$$

# Nozières and Schmitt-Rink (NSR) Theory

P. Nozières and S. Schmitt-Rink, J. Low Temp. Phys., **59**, 195 (1985).

- Contribution to thermodynamic potential by pairing-fluctuation

$$\Omega - \Omega_f = T \sum_{\mathbf{q}} \sum_{\omega_s} \log [1 - \chi(\mathbf{q}, \omega_s)]$$



- Density correction  $(N - N_f) = -\frac{\partial}{\partial \mu} (\Omega - \Omega_f)$

- Equivalent to integrating Gaussian fluctuations.

C. A. R. Sá de Melo, M. Randeria, and J. R. Engelbrecht, Phys. Rev. Lett., **71**, 3202,(1993).

- Recent developments

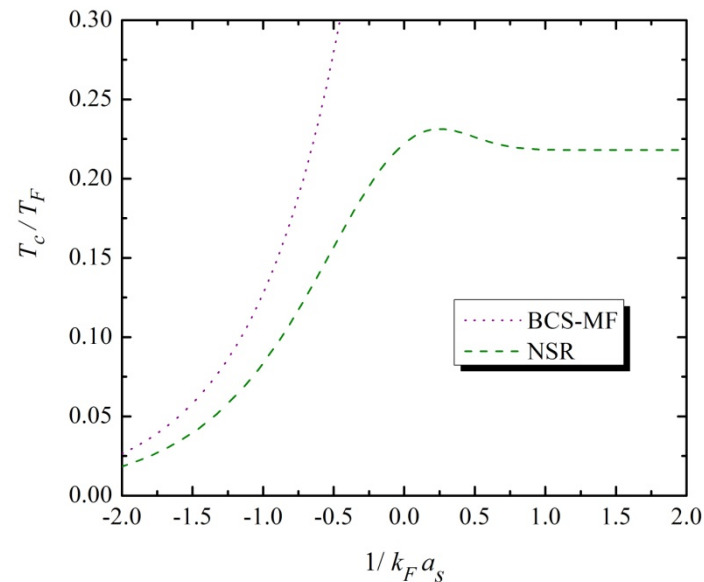
K. Levin *et al.*, arXiv:0810.1938.

H. Hu, X.-J. Liu and P. D. Drummond, Phys. Rev. A, **77**, 061605(R) (2008)

# Nozieres and Schmitt-Rink (NSR) Theory

P. Nozières and S. Schmitt-Rink, J. Low Temp. Phys., **59**, 195 (1985).

- Recovering  $T_c$  of ideal BEC  $T_c^{(\text{BEC})} = 0.218T_F$



- Molecule scattering length  $a_b \approx 0.55a_s$  (mean-field  $a_b = 2a_s$ ),

H. Hu, X.-J. Liu and P. D. Drummond, Europhys. Lett. **74**, 574 (2006)

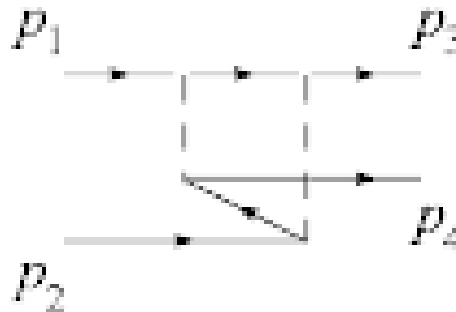
due to vacuum renormalization.

K. Huang, Z.-Q. Yu, and L. Yin, Phys. Rev. A **79**, 053602 (2009)

# Induced interaction in the BCS limit

L. P. Gorkov and T. K. Melik-Barkhudarov, Sov. Phys. JETP **13**, 1018 (1961)

The induced interaction is generated by the medium

$$U_{\text{ind}}(\mathbf{p}_1 - \mathbf{p}_4) = g^2 \chi(\mathbf{p}_1 - \mathbf{p}_4) =$$


Effective pairing interaction

$$U_{\text{eff}}(\mathbf{p}) = g + U_{\text{ind}}(\mathbf{p}) = g + g^2 \chi(\mathbf{p}),$$

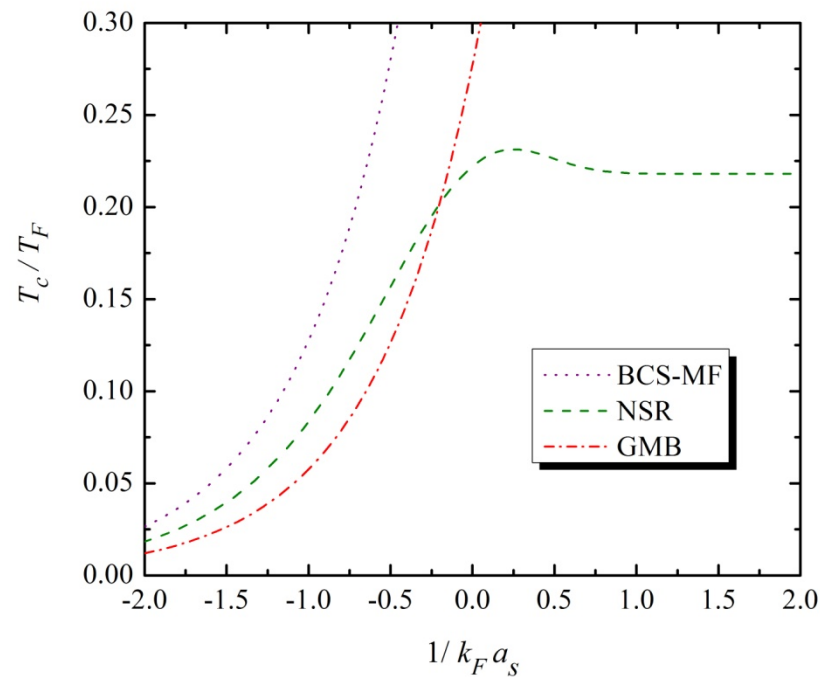
$$\langle U_{\text{eff}} \rangle^{-1} \approx g^{-1} - \langle \chi \rangle = g^{-1} - \frac{\ln 4e}{3} N(\varepsilon_F),$$

# Induced interaction in the BCS limit

L. P. Gorkov and T. K. Melik-Barkhudarov, Sov. Phys. JETP **13**, 1018 (1961)

In the BCS limit  $T_c$  is reduced by factor 2.22

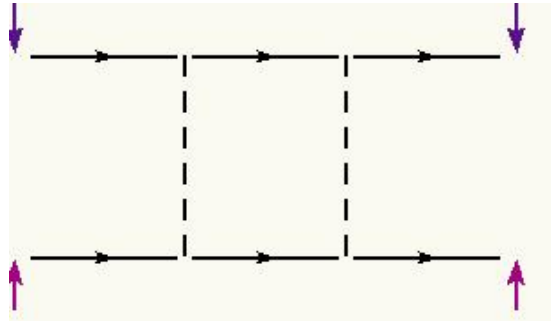
$$T_c^{(\text{GMB})} \approx T_c^{(\text{BCS})} / (4e)^{1/3} \approx 0.28 T_F \exp[\pi / (2k_F a_s)]$$



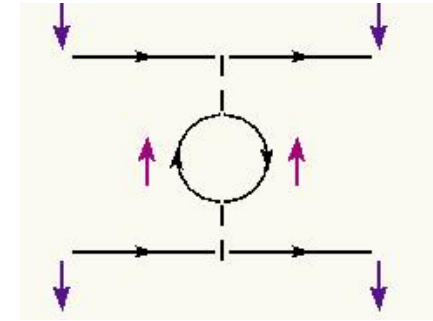
Unifying NSR and GMB results?

## (2) Induced interaction for BEC-BCS crossover

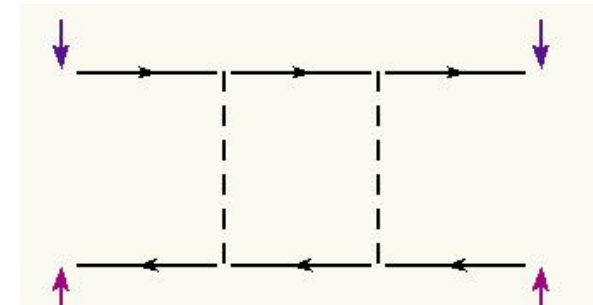
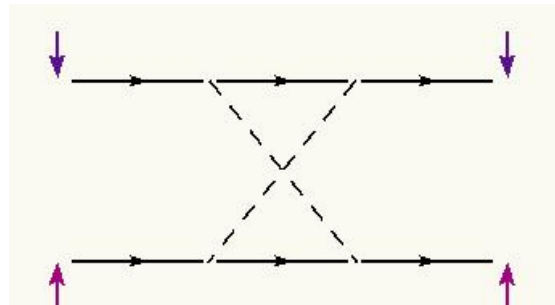
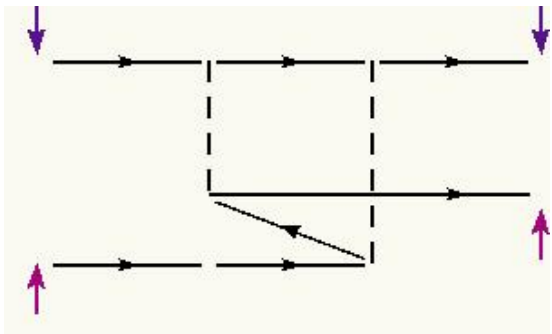
### 2<sup>nd</sup> order processes



Particle-particle fluctuation, part of  $t$ -matrix



Spin-triplet interaction



Induced interaction is generated by particle-hole fluctuation.

## Induced interaction for BEC-BCS crossover (RPA)



$$U_{\text{ind}}(\mathbf{k}', \omega) = \frac{g^2 \chi(\mathbf{k}', \omega)}{1 - g \chi(\mathbf{k}', \omega)}$$

Effective pairing interaction  $U_{\text{eff}}(\mathbf{k}', \omega) = g + U_{\text{ind}}(\mathbf{k}', \omega) = \frac{g}{1 - g \chi(\mathbf{k}', \omega)}$

S-wave component  $g' = \frac{g}{1 - g \langle \chi \rangle}$

The average is taken at zero frequency and Fermi surface,

$$\langle \chi \rangle = \frac{m}{4\pi^2 \hbar^2} \int_{-1}^1 d \cos \theta \int_0^\infty dk \frac{k}{k'} f_k \ln \left| \frac{k' - 2k}{k' + 2k} \right|, \quad k' = |\mathbf{k}_1 - \mathbf{k}_4| = k_F \sqrt{2(1 + \cos \theta)},$$

or at  $\mathbf{k}'=0$ ,  $\langle \chi \rangle = -\frac{m}{2\pi^2 \hbar^2} \int_0^\infty dk f_k.$

# Induced interaction for BEC-BCS crossover (RPA)

BCS limit

$$g'^{-1} = g^{-1} - \frac{\ln 4e}{3} N(\varepsilon_F),$$

Consistent with GMB result

BEC limit

$$g' \simeq g, \quad \text{due to } \langle \chi \rangle \simeq 0$$

Induced interaction is unimportant in the BEC limit, because particle-hole fluctuation is suppressed by the gap in single-particle excitations.

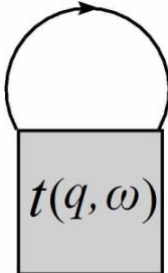
### (3) Effect on superfluid transition temperature $T_c$

Consider pairing fluctuation in  $t$ -matrix  
with induced interaction included



$T_c$  determined by Thouless criterion

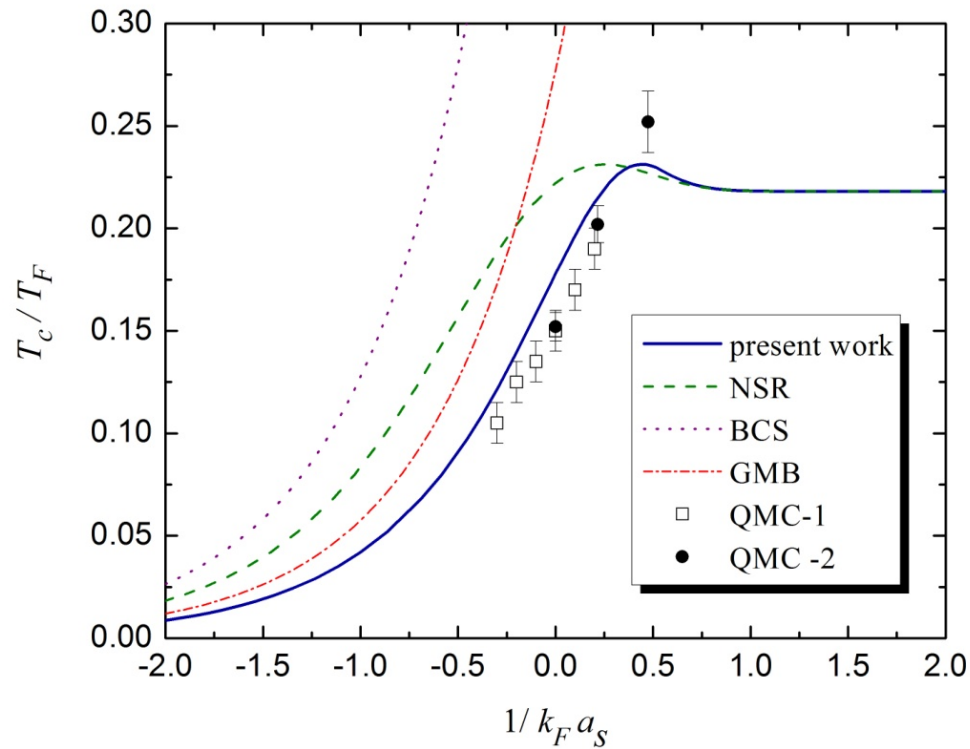
$$t^{-1}(0,0) = 0$$

Density correction due to self-energy  $\Sigma =$  

$$\delta n = \frac{2}{\beta V} \sum_q \mathcal{G}_0^2(q) \Sigma(q)$$

( In BEC limit, density correction is due to thermal molecule density and vacuum renormalization to molecular condensate. )

# T<sub>c</sub> of BEC-BCS crossover



T<sub>c</sub> at unitarity is smaller than NSR T<sub>c</sub> by about 20%.

Induced interaction is important on BCS side and in unitary region.

(Z.-Q. Yu, K. Huang, and L. Yin, PRA **79**, 053636 (2009))

## Tc at unitarity

Ours [1]	$0.178 T_F$
NSR [2]	$0.222 T_F$
Self-consistent NSR [3]	$0.160 T_F$
Pseudogap Theory [4]	$0.260 T_F$
RG [5]	$0.13 T_F$
RG [6]	$0.264 T_F$
Quantum Monte-Carlo [7,8]	$0.15(1) T_F$

[1] Z.-Q. Yu, K. Huang, and L. Yin, Phys. Rev. A 79, 053636 (2009).

[2] P. Nozieres and S. Schmitt-Rink, J. Low Temp. Phys. 59, 195 (1985).

[3] R. Haussmann *et al.*, Phys. Rev. A. 75, 023610 (2007).

[4] Q. Chen *et al.*, Phys. Rep. 412, 1 (2005).

[5] K. B. Gubbels and H. T. C. Stoof, Phys. Rev. Lett. 100, 140407 (2008).

[6] S. Floerchinger *et al.*, Phys. Rev. B 78, 174528 (2008).

[7] E. Burovski *et al.*, Phys. Rev. Lett. 101, 090402 (2008).

[8] A. Bulgac, J. E. Drut and P. Magierski, Phys. Rev. A 78, 023625 (2008).

# Conclusion

- The induced interaction plays an important role in BCS and unitary region.
- A successful microscopic theory for BEC-BCS crossover should take into account fluctuations in both particle-particle and particle-hole channel.
- Problems to be solved in our approach: self consistency, broken symmetry state...