pNRQCD: an EFT for heavy quarkonium

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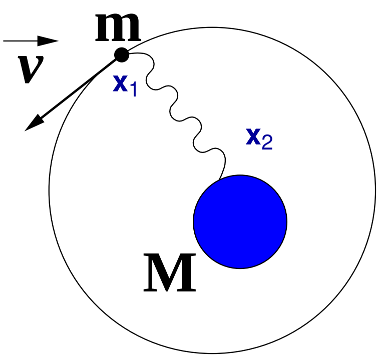
Outline

pNRQCD: FORMALISM

PHENOMENOLOGICAL ANALYSIS

CONCLUSIONS

$v \sim \alpha \ll 1$



$$\mathbf{x} = \mathbf{x}_1 - \mathbf{x}_2 \qquad \mathbf{X} = \frac{m}{m+M}\mathbf{x}_1 + \frac{M}{m+M}\mathbf{x}_2$$

$$H = \frac{\mathbf{p}^2}{2m} + V(r) \qquad V(r) = -\frac{Z_1 Z_2 \alpha}{r}$$

Scales: hard, soft, ultrasoft; $m \gg mv \gg mv^2$...

Introduction

Are we ready to describe new and old data on heavy quarkonium physics?

$\rightarrow$ We have an effective field theory, Potential Non-Relativistic QCD, which describes the heavy quarkonium dynamics in the weak and strong coupling situation. $m \gg mv \gg mv^2$

$$\left.\begin{array}{l} \left(i\partial_0 - \dfrac{\mathbf{p}^2}{2m} - V_s^{(0)}(r)\right)\Phi(\mathbf{r}) = 0 \\ +\text{corrections to the potential} \\ +\text{interaction with other low} \\ \quad \text{energy degrees of freedom} \end{array}\right\} \text{potential NRQCD} \qquad E \sim mv^2$$

In the weak coupling regime the starting point is $V_s^{(0)} = -C_f \dfrac{\alpha_s}{r}$.
In the strong coupling regime case

$$V_s^{(0)}(\mathbf{r}) = \lim_{T\to\infty} \frac{i}{T} \log \langle W_\Box \rangle \quad \text{Wilson, Susskind}$$

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Our aim is to provide a systematic method to deal with NR bound state systems. We will introduce a hierarchy of EFTs when sequentially integrating out each scale (only one scale in each step, strong simplification).

QCD

↓ Integrating out the hard scale (m)

NRQCD — Caswell, Lepage

↓ Integrating out the soft scale (mv)

pNRQCD — Soto, Pineda

1) Matching QCD to NRQCD. Integrating out the hard scale, m

NRQCD has an ultraviolet cutoff Λ such that $m \gg \Lambda$ and larger than any other dynamical scale in the problem. $\Psi = \psi + \chi$

$$\mathcal{L}_{NRQCD} = \bar{\Psi} i\gamma^0 D_0 \Psi + \bar{\Psi}\left\{ \frac{\mathbf{D}^2}{2m} + c_F\, g \frac{\mathbf{\Sigma}\cdot\mathbf{B}}{2m} + c_D\, g \frac{\gamma^0 (\mathbf{D}\cdot\mathbf{E} - \mathbf{E}\cdot\mathbf{D})}{8m^2} \right.$$

$$\left. + i c_S\, g \frac{\gamma^0 \mathbf{\Sigma}\cdot(\mathbf{D}\times\mathbf{E} - \mathbf{E}\times\mathbf{D})}{8m^2} + \frac{\mathbf{D}^4}{8m^3} \right\}\Psi$$

$$-\frac{1}{4} c_1 F_{\mu\nu}F^{\mu\nu} + \frac{c_2}{m^2} gF_{\mu\nu}D^2 gF^{\mu\nu} + \frac{c_3}{m^2} g^3 f_{ABC} F^A_{\mu\nu} F^B_{\mu\alpha} F^C_{\nu\alpha}$$

$$\delta\mathcal{L}_{NRQCD} = \frac{d_{ss}}{m_1 m_2}\psi_1^\dagger\psi_1\chi_2^\dagger\chi_2 + \frac{d_{sv}}{m_1 m_2}\psi_1^\dagger\boldsymbol{\sigma}\psi_1\chi_2^\dagger\boldsymbol{\sigma}\chi_2$$

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Lepage, Caswell, Thacker

$$c_i \sim 1 + \alpha_s\left(A\log\frac{m}{\mu} + B\right) \qquad d_i \sim \alpha_s\left(1 + \alpha_s\left(A\log\frac{m}{\mu} + B\right)\right)$$

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1) Matching QCD to NRQCD. Integrating out the hard scale, m
Relativistic Feynman diagrams ←
In order to integrate the mass scale it is only needed

$$m >> |\mathbf{p}|, E, \Lambda_{QCD}$$

One matches to HQET from a practical point of view.
Analytical expansion over the three-momentum and residual energy in the integrand before the integration is made in both the full and the effective theory.

QCD

$$\int d^4q f(q, m, |\mathbf{p}|, E) = \int d^4q f(q, m, 0, 0) + O\left(\frac{E}{m}, \frac{|\mathbf{p}|}{m}\right)$$

NRQCD

$$\int d^4q f(q, |\mathbf{p}|, E) = \int d^4q f(q, 0, 0) = 0\,!!$$

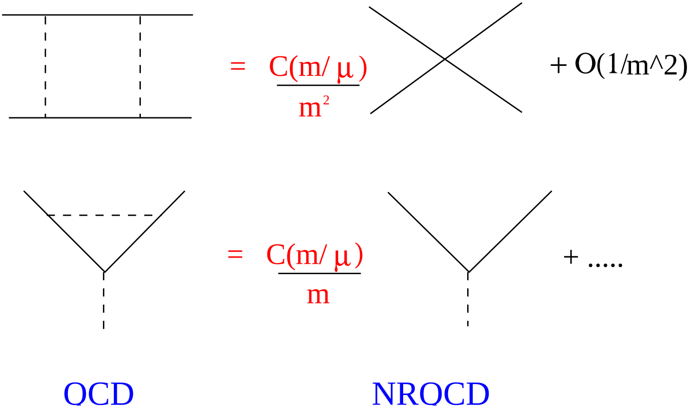
Dimensional regularization. The computation of loops in the effective theory just gives zero.

1) Matching QCD to NRQCD. Integrating out the hard scale, m

Relativistic Feynman diagrams ←

Final rule:

- One matches loops in QCD with only one scale (the mass) to tree level diagrams in NRQCD.



Manohar; Soto, Pineda

1) Matching QCD to NRQCD. Integrating out the hard scale, m
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2) Matching NRQCD to pNRQCD. Integrating out the soft scale, mv
2a) Weak coupling ($mv \gg \Lambda_{QCD}$).
Power counting/scales
Scales: m, p, $1/r$, $\Lambda_{mp} = \{\Lambda_{QCD}, mv^2, ...\}$
Dimensionless quantities:

$$\frac{p}{m}, \ \alpha_s, \ \frac{1}{m\,r}, \ \boxed{\Lambda_{mp} r} \ll 1$$

The multipole expansion can be used in the new EFT.

$$L_{pNRQCD} = L'_{NRQCD} + \int\int d^3x_1 d^3x_2 \psi(x_1)\chi_c(x_2) V(x_1 - x_2)\psi^\dagger(x_1)\chi_c^\dagger(x_2)$$

L'_{NRQCD}, gluons multipole expanded (only ultrasoft gluons).
To go to the wave function description one has to project to the quark-antiquark sector.

$$\int d^3\mathbf{x}_1 d^3\mathbf{x}_2 \Psi(\mathbf{x}_1, \mathbf{x}_2)\psi(x_1)\chi_c(x_2)|0\rangle$$

$$H \int d^3\mathbf{x}_1 d^3\mathbf{x}_2 \Psi(\mathbf{x}_1, \mathbf{x}_2)\psi^\dagger(\mathbf{x}_1)\chi_c^\dagger(\mathbf{x}_2)|0\rangle = \int d^3\mathbf{x}_1 d^3\mathbf{x}_2 (\hat{h}\Psi(\mathbf{x}_1, \mathbf{x}_2))\psi^\dagger(\mathbf{x}_1)\chi_c^\dagger(\mathbf{x}_2)|0\rangle$$

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For QED

$$L_{pNRQED} = \int d^3\mathbf{x}_1 d^3\mathbf{x}_2 \Psi^\dagger(\mathbf{x}_1,\mathbf{x}_2)\left(iD_0 + \frac{\mathbf{D}^2_{\mathbf{x}_1}}{2m_1} + \frac{\mathbf{D}^2_{\mathbf{x}_2}}{2m_2} - V(\mathbf{x},\mathbf{p})\right)\Psi(\mathbf{x}_1,\mathbf{x}_2)$$

$$= \int d^3\mathbf{x}_1 d^3\mathbf{x}_2 \Psi^\dagger(\mathbf{x}_1,\mathbf{x}_2)\Big(i\partial_0 + \frac{\boldsymbol{\nabla}^2_{\mathbf{x}}}{m} + \frac{\boldsymbol{\nabla}^2_{\mathbf{X}}}{4m} - e\mathbf{x}\cdot\boldsymbol{\nabla} A_0(\mathbf{X}) - 2ie\frac{\mathbf{A}(\mathbf{X})\cdot\boldsymbol{\nabla}_{\mathbf{x}}}{m} - V(\mathbf{x},\mathbf{p})\Big)\Psi(\mathbf{x}_1,\mathbf{x}_2)$$

New fields: Singlet S, Octet (O) and US gluons.
Gauge transformation:

$$S(\mathbf{x},\mathbf{X},t) \to S(\mathbf{x},\mathbf{X},t)\,, \qquad O(\mathbf{x},\mathbf{X},t) \to g(\mathbf{X},t)O(\mathbf{x},\mathbf{X},t)g^{-1}(\mathbf{X},t)\,.$$

Field Redefinitions.

$$\Psi(\mathbf{x}_1,\mathbf{x}_2) = \phi(\mathbf{x}_1,\mathbf{x}_2)S(\mathbf{x},\mathbf{X}) + \phi(\mathbf{x}_1,\mathbf{X})O(\mathbf{x},\mathbf{X})\phi(\mathbf{X},\mathbf{x}_2)$$

$$\phi(\mathbf{y},\mathbf{x},t) \equiv \mathrm{P}\exp\left\{ig\int_0^1 ds\,(\mathbf{y}-\mathbf{x})\cdot\mathbf{A}(\mathbf{x}-s(\mathbf{x}-\mathbf{y}),t)\right\}$$

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2) Matching NRQCD to pNRQCD. Integrating out the soft scale, mv

2a) Weak coupling ($mv \gg \Lambda_{QCD}$). Interpolating fields: $3 \otimes 3^* = 1 \oplus 8$

$$Q_2^\dagger(\mathbf{x}_2,t)\phi(\mathbf{x}_2,\mathbf{x}_1;t)Q_1(\mathbf{x}_1,t) = Z_s^{1/2}(\mathbf{x})S(\mathbf{X},\mathbf{x},t) + \cdots$$

$$Q_2^\dagger(x_2)\phi(\mathbf{x}_2,\mathbf{X};t)T^a\phi(\mathbf{X},\mathbf{x}_1;t)Q_1(x_1) = Z_o^{1/2}(\mathbf{x})O^a(\mathbf{X},\mathbf{x},t) + \cdots$$

pNRQCD Lagrangian at $O(r)$ ($r \sim 1/(mv)$; $V_s^{(0)}, gA^\mu(X) \sim mv^2$)

$$\begin{aligned}
\mathcal{L}_{pNRQCD} &= \mathrm{Tr}\left\{ S^\dagger \left(i\partial_0 - V_s^{(0)}(\mathbf{x})\right) S - S^\dagger \left(\frac{\mathbf{p}^2}{m} + \sum_n \frac{V_s^{(n)}(\mathbf{x})}{m^n}\right) S \right\} \\
&+ \mathrm{Tr}\left\{ O^\dagger \left(iD_0 - V_o^{(0)}(\mathbf{x})\right) O - O^\dagger \left(\frac{\mathbf{p}^2}{m} + \sum_n \frac{V_o^{(n)}(\mathbf{x})}{m^n}\right) O \right\} \\
&+ gV_A(\mathbf{x})\mathrm{Tr}\left\{ O^\dagger \mathbf{x}\cdot\mathbf{E}\, S + S^\dagger \mathbf{x}\cdot\mathbf{E}\, O \right\} \\
&+ g\frac{V_B(\mathbf{x})}{2}\mathrm{Tr}\left\{ O^\dagger \mathbf{x}\cdot\mathbf{E}\, O + O^\dagger O \mathbf{x}\cdot\mathbf{E} \right\}.
\end{aligned}$$

$$V = V(c(\nu_s/m), d(\nu_s/m, \nu_p/m), r, \nu_s, \nu_{us}) \sim \sum_n c_n(\nu; m, r)\alpha_s^n(\nu)$$

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Relativistic Feynman diagrams $\leftarrow$

2) Matching NRQCD to pNRQCD. Integrating out the soft scale, mv

2a) Weak coupling ($mv \gg \Lambda_{QCD}$). Interpolating fields: $3 \otimes 3^* = 1 \oplus 8$

$$Q_2^\dagger(\mathbf{x}_2, t)\phi(\mathbf{x}_2, \mathbf{x}_1; t)Q_1(\mathbf{x}_1, t) = Z_s^{1/2}(\mathbf{x})S(\mathbf{X}, \mathbf{x}, t) + \cdots$$

$$Q_2^\dagger(x_2)\phi(\mathbf{x}_2, \mathbf{X}; t)T^a\phi(\mathbf{X}, \mathbf{x}_1; t)Q_1(x_1) = Z_o^{1/2}(\mathbf{x})O^a(\mathbf{X}, \mathbf{x}, t) + \cdots$$

pNRQCD Lagrangian at $O(r)$ ($r \sim 1/(mv)$; $V_s^{(0)}, gA^\mu(X) \sim mv^2$)

$$\begin{aligned}
\mathcal{L}_{pNRQCD} &= \mathrm{Tr}\left\{ S^\dagger \left(i\partial_0 - V_s^{(0)}(\mathbf{x}) \right) S - S^\dagger \left(\frac{\mathbf{p}^2}{m} + \sum_n \frac{V_s^{(n)}(\mathbf{x})}{m^n} \right) S \right\} \\
&+ \mathrm{Tr}\left\{ O^\dagger \left(iD_0 - V_o^{(0)}(\mathbf{x}) \right) O - O^\dagger \left(\frac{\mathbf{p}^2}{m} + \sum_n \frac{V_o^{(n)}(\mathbf{x})}{m^n} \right) O \right\} \\
&+ gV_A(\mathbf{x}) \mathrm{Tr}\left\{ O^\dagger \mathbf{x} \cdot \mathbf{E}\, S + S^\dagger \mathbf{x} \cdot \mathbf{E}\, O \right\} \\
&+ g\frac{V_B(\mathbf{x})}{2} \mathrm{Tr}\left\{ O^\dagger \mathbf{x} \cdot \mathbf{E}\, O + O^\dagger O \mathbf{x} \cdot \mathbf{E} \right\}.
\end{aligned}$$

$$V = V(c(\nu_s/m), d(\nu_s/m, \nu_p/m), r, \nu_s, \nu_{us}) \sim \sum_n c_n(\nu; m, r)\alpha_s^n(\nu)$$

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2a) Weak coupling ($mv \gg \Lambda_{QCD}$). Potential= HQET-like Feynman diagrams $\leftarrow$

Same idea than in NRQCD. Expansion in the scales that are left in the effective theory. We integrate out the scale k (transfer momentum between the quark and antiquark).
Analytical expansion of **1/m** (and therefore **p**) and **E** before the integration is made in both the full and the effective theory. Effectively HQET-like rules (HQET quark propagator).
NRQCD

$$\int d^4 q f(q,k,|\mathbf{p}|,E) = \int d^4 q f(q,k,0,0) + O\left(\frac{E}{k},\frac{|\mathbf{p}|}{k}\right) \qquad \text{potentials}$$

pNRQCD

$$\int d^4 q f(q,|\mathbf{p}|,E) = \int d^4 q f(q,0,0) = 0\,!!$$

Dimensional regularization. The computation in the effective theory just gives zero.

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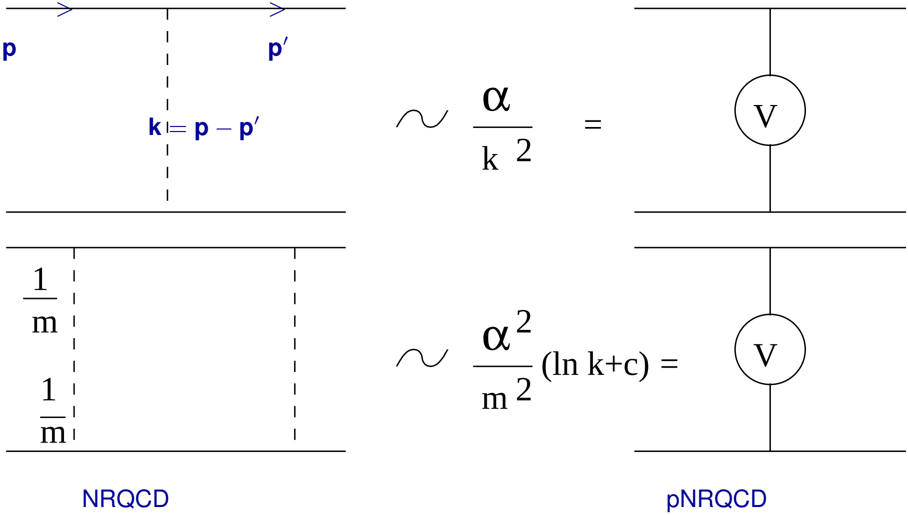

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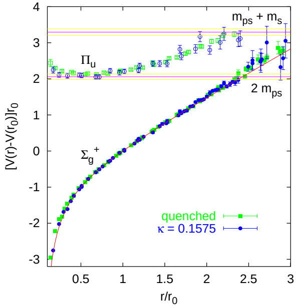
2b) Strong coupling. Interpolating fields: S, O and soft gluons $\rightarrow$ S



Matching scale $\nu_{us} \ll \Lambda_{QCD}$. Coloured-like degrees of freedom decouple. Mass gap of hybrids and glueballs of $O(\Lambda_{QCD} \sim mv) \gg mv^2$.

$r_0 \simeq 0.5$ fm, hep-lat/0003012.

- Pure QCD (no light fermions): the singlet (S)
- QCD: singlet plus pions (non-potential effects).

pNRQCD Lagrangian

$$\mathcal{L}_{pNRQCD} = \mathrm{Tr}\left\{ S^\dagger \left(i\partial_0 - V_s^{(0)}(\mathbf{x}) \right) S - S^\dagger \left(\frac{\mathbf{p}^2}{m} + \sum_n \frac{V_s^{(n)}(\mathbf{x})}{m^n} \right) S \right\}.$$

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$$\langle 0|Q_2^\dagger(x_2)\phi(x_2,x_1)Q_1(x_1)Q_1^\dagger(y_1)\phi(y_1,y_2)Q_2(y_2)|0\rangle,$$

NRQCD

$$\delta^3(\mathbf{x}_1-\mathbf{y}_1)\delta^3(\mathbf{x}_2-\mathbf{y}_2)\langle W_\Box\rangle,$$

pNRQCD

$$Z_s(\mathbf{r})\delta^3(\mathbf{x}_1-\mathbf{y}_1)\delta^3(\mathbf{x}_2-\mathbf{y}_2)e^{-iTV_s^{(0)}(\mathbf{r})}$$

One obtains:

$$V_s^{(0)}(\mathbf{r}) = \lim_{T\to\infty}\frac{i}{T}\log\langle W_\Box\rangle = -C_f\frac{\alpha_s}{r}+O(\alpha_s^2)$$

Wilson, Susskind

$$V_s^{(1)} = -\lim_{T\to\infty}\int_0^T dt\,\frac{t}{2}\,\langle\langle g\mathbf{E}_1(t)\cdot g\mathbf{E}_1(0)\rangle\rangle_c = -C_f C_A\frac{\alpha_s^2}{4r^2}+O(\alpha_s^3)$$

Brambilla, Soto, Pineda, Vairo
and $O(1/m^2)$ ...

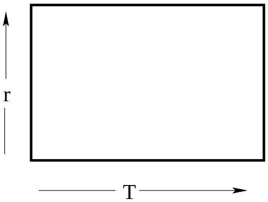
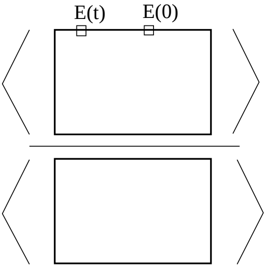


Figure: *Graphic representation of the static Wilson loop. We adopt the convention that the time propagates from the left to the right. Therefore, horizontal lines correspond to the quark trajectories and the vertical lines to the Schwinger strings.*



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$$G_s(E) = P_s \frac{1}{h_s^{(0)} - H_I - E} P_s = G_s^{(0)} + \delta G_s \qquad G_s^{(0)}(E) = \frac{1}{h_s^{(0)} - E}$$

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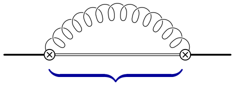
A) Ultrasoft loops (only at weak coupling: lamb shift-like): $\mathbf{x} \cdot \mathbf{E} \leftarrow$



$1/(E - V_o^{(0)} - \mathbf{p}^2/m)$

$$\delta G_s \sim \frac{1}{h_s^{(0)} - E} \int \frac{d^3\mathbf{k}}{(2\pi)^{D-1}} \mathbf{r} \frac{k}{k + h_o^{(0)} - E} \mathbf{r} \frac{1}{h_s^{(0)} - E}$$

$$\sim \frac{1}{h_s^{(0)} - E} \mathbf{r} (h_o^{(0)} - E)^3 \left\{ \frac{1}{\epsilon} + \gamma + \ln \frac{(h_o^{(0)} - E)^2}{\mu_{us}^2} + C \right\} \mathbf{r} \frac{1}{h_s^{(0)} - E}$$

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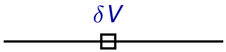
B) Quantum mechanics perturbation theory (both at weak and strong coupling)$\leftarrow$

$$\delta G_s \sim \frac{1}{h_s^{(0)} - E} \delta V_s \frac{1}{h_s^{(0)} - E} + \frac{1}{h_s^{(0)} - E} \delta V_s \frac{1}{h_s^{(0)} - E} \delta V_s \frac{1}{h_s^{(0)} - E} + \cdots$$

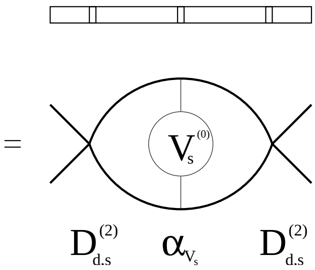


Ultraviolet divergences are governed by the short-distance behavior of the potentials, i.e. by perturbation theory. Therefore, they can be computed and absorbed in the matching coefficients of the currents or in the potentials.

Example:

$$\delta(r)\frac{1}{h_s^{(0)}-E}\delta(r) \to \delta(r)\frac{1}{\mathbf{p}^2/m-E}\frac{C_f\alpha_s}{r}\frac{1}{\mathbf{p}^2/m-E}\delta(r)$$

Since the singular behavior of the potential loops appears for $\mathbf{p}^2/m \gg \alpha_s/r$, a perturbative expansion in α_s is licit. The divergences can be absorbed in the matching coefficients of the local potentials (proportional to the $\delta^{(3)}(\mathbf{r})$) providing with the renormalization group equations.



$$\langle \mathbf{r}=0|\frac{1}{E-\mathbf{p}^2/m}C_f\frac{\alpha_{V_s}}{r}\frac{1}{E-\mathbf{p}^2/m}|\mathbf{r}=0\rangle$$

$$\sim \int\frac{\mathrm{d}^dp'}{(2\pi)^d}\int\frac{\mathrm{d}^dp}{(2\pi)^d}\frac{m}{\mathbf{p}'^2-mE}C_f\frac{4\pi\alpha_{V_s}}{\mathbf{q}^2}\frac{m}{\mathbf{p}^2-mE} \sim -C_f\frac{m^2\alpha_{V_s}}{16\pi}\frac{1}{\epsilon},$$

where $D=4+2\epsilon$ and $\mathbf{q}=\mathbf{p}-\mathbf{p}'$. This divergence is absorbed in $D_{d,s}^{(2)}$.

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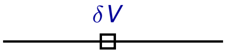

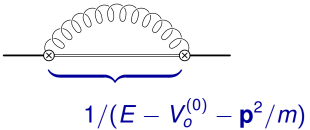

Phenomenological analysis:

- Which states belong to the weak/strong coupling regime?
  To which extent the static potential can be described with perturbation theory
  Sumino; Pineda; Sumino, Recksiegel; Lee, ...

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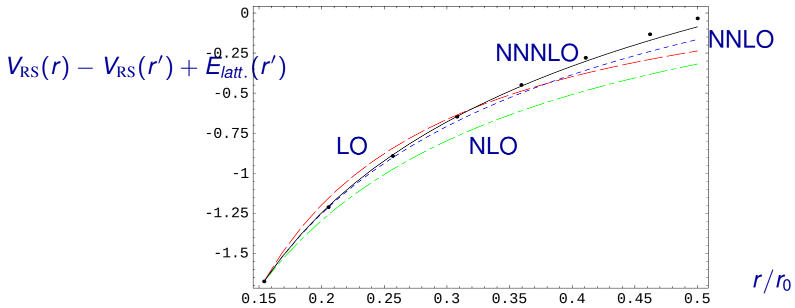
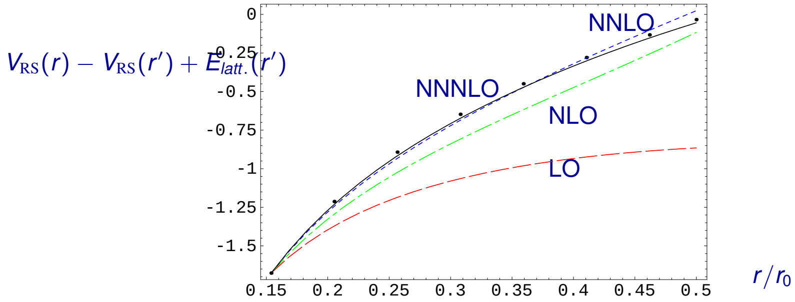


V_RS(r) − V_RS(r′) + E_latt.(r′)
0
-0.25
-0.5
-0.75
-1
-1.25
-1.5
NNNLO
NNLO
LO
NLO
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
r/r0



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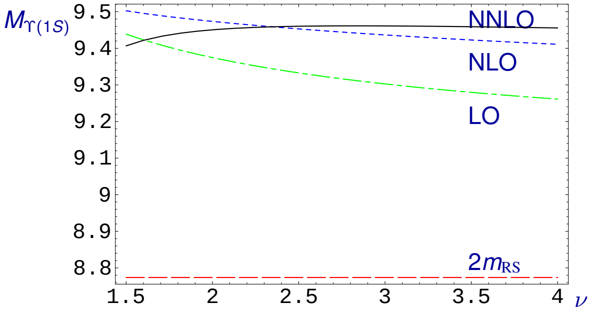
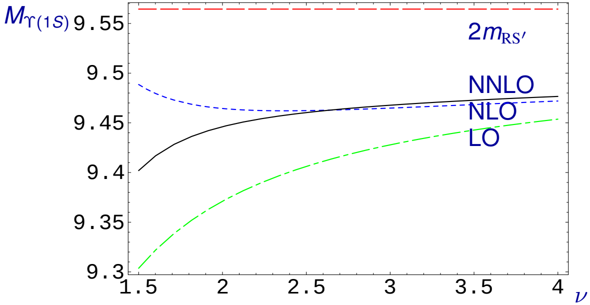

$M_{\Upsilon(1S)}$
9.5
9.4
9.3
9.2
9.1
9
8.9
8.8
NNLO
NLO
LO
$2m_{RS}$
1.5
2
2.5
3
3.5
4
ν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9.55
9.5
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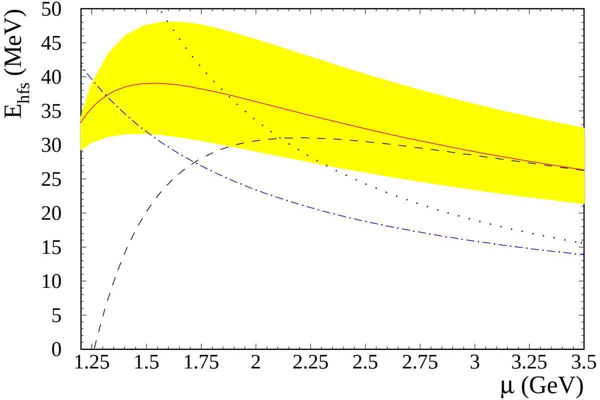
Kniehl, Penin, Smirnov, Steinhauser, Pineda;
Penin, Smirnov, Steinhauser, Pineda

$$\begin{aligned}\delta E &\sim m\alpha^4 + m\alpha^5 \ln\alpha + m\alpha^6 \ln^2\alpha + \cdots \\ &+ m\alpha^5 + m\alpha^6 \ln\alpha + m\alpha^7 \ln^2\alpha + m\alpha^8 \ln^3\alpha + \cdots\end{aligned}$$



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B_c mass (MeV)

State	1^1S_0
Experiment	$6287 \pm 4.8 \pm 1.1$
lattice04	6304(16)
BV00	6326(29)
BSV01	6324(22)
BSV02	6307(17)

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  Brambilla, Vairo; Brambilla, Sumino, Vairo
  Higher excitations of bottomonium and charmonium. Not conclusive. Different claims by
  Brambilla, Sumino, Vairo and Sumino, Recksiegel versus Beneke, Kiyo, Schuler
  Prediction of the $\eta_c(2S)$.

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  η_b
  Sumino, Recksiegel; Kniehl, Penin, Pineda, Smirnov, Steinhauser (with Renormalization group).
  Problems with the recent experimental determination:
  theory $\sim$ 40 Mev versus experiment $\sim$ 70 MeV
  B_c
  Brambilla, Vairo; Brambilla, Sumino, Vairo
  Higher excitations of bottomonium and charmonium. Not conclusive. Different claims by
  Brambilla, Sumino, Vairo and Sumino, Recksiegel versus Beneke, Kiyo, Schuler
  Prediction of the $\eta_c(2S)$.

Phenomenological analysis:

- Which states belong to the weak/strong coupling regime?
  To which extent the static potential can be described with perturbation theory
  Sumino; Pineda; Sumino, Recksiegel; Lee; Bali, Pineda; ...
- Spectroscopy at weak coupling
- Spectroscopy at strong coupling
  Determination of the potentials: Bali, Schilling, Wachter; Koma, Koma, Wittig.
  Lacking the rest of potentials, simulations with dynamical fermions, control of the scheme dependence.
  Some old analysis by Bali, Schilling and Wachter

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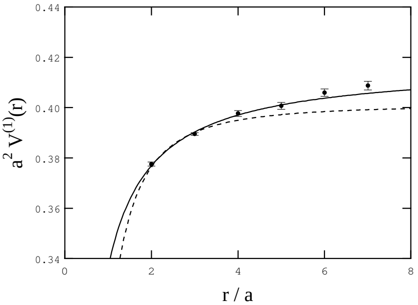

Figure: $V^{(1)}$ *from Koma, Koma and Wittig, hep-lat/0607009.*

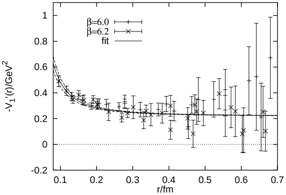

Figure: *The spin-orbit potential $-V_1'$ with the fit $\sigma + h/r^2$ from Bali, Schilling and Wachter, 1997. The lattice simulations are quenched. The fitting parameters are $\sigma \approx (468\ MeV)^2$ and $h \approx 0.067$.*

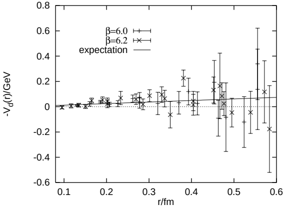

Figure: *The potential V_d together with the curve $-\sigma/9\,r$, from Bali, Schilling and Wachter, 1997. The lattice simulations are quenched. The fitting parameters are $\sigma \approx (468\ MeV)^2$ and $h \approx 0.067$.*

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Vacuum polarization in the non-relativistic limit

$$J^{\mu} = \bar{Q}\gamma^{\mu}Q = c_1\psi^{\dagger}\boldsymbol{\sigma}\chi + \cdots\,, \qquad c_1 = 1 + a_1\alpha_s + a_2\alpha_s^2 + \cdots$$

c_1 at NNLO: Hoang(QED); Beneke, Signer, Smirnov; Czarnecki, Melnikov
c_1, c_0 at NLL: Pineda; Hoang, Stewart
c_1/c_0 at NNLL: Penin, Pineda, Smirnov, Steinhauser
c_1, c_0 at NNLL (partial): Pineda, Signer

$$(q_{\mu}q_{\nu} - g_{\mu\nu})\Pi(q^2) = i\int d^4x e^{iqx}\langle \mathrm{vac}|J_{\mu}(x)J_{\nu}(0)|\mathrm{vac}\rangle$$

$$\Pi(q^2) \sim c_1^2\langle \mathbf{r}=\mathbf{0}|\frac{1}{E-H}|\mathbf{r}=\mathbf{0}\rangle$$

$$G(0,0,E) = \sum_{m=0}^{\infty}\frac{|\phi_{0m}(0)|^2}{E_{0m} - E + i\epsilon - i\Gamma_t} + \frac{1}{\pi}\int_0^{\infty} dE'\,\frac{|\phi_{0E'}(0)|^2}{E_{0E'} - E + i\epsilon - i\Gamma_t}$$

$M(V_Q(nS))$ is also needed in order to obtain expressions for the t-$\bar{t}$ production near threshold with NNLL accuracy:
$M(V_Q(nS))$ at NNLL: Pineda; Hoang, Stewart

Relation of the vacuum polarization with $\sigma_{t\bar{t}}$, non-relativistic sum rules and $\Gamma(V_Q(nS) \rightarrow e^+e^-)$

Determination of m_b, m_t, α_s, Higgs-top yukawa coupling, ...

$$\Gamma(V \rightarrow e^+e^-) \sim \frac{1}{m^2} c_1^2 |\phi(\mathbf{0})|^2$$

$$|\phi_n(\mathbf{0})|^2 = \left|\phi_n^{(0)}(\mathbf{0})\right|^2 (1 + \delta\phi_n) = \underset{E=E_n}{\mathrm{Res}} G(\mathbf{0}, \mathbf{0}; E),$$

Note that $|\phi_n(\mathbf{0})|^2$ is SCHEME and SCALE dependent.

$$\sigma_{t-\bar{t}} \sim c_1(\nu)^2 \mathrm{Im} G(0, 0, \sqrt{s}) + \cdots$$

$$M_n \equiv \frac{12\pi^2 e_b^2}{n!} \left(\frac{d}{dq^2}\right)^n \Pi(q^2)\Big|_{q^2=0} = \int_0^\infty \frac{ds}{s^{n+1}} R_{b\bar{b}}(s),$$

$$M_n = 48\pi e_b^2 N_c \int_{-\infty}^{\infty} \frac{dE}{(E + 2m_b)^{2n+3}} \left(c_1^2 - c_1 d_1 \frac{E}{3m_b}\right) \mathrm{Im}\, G(0, 0, E)$$

Inclusive electromagnetic decays: bottomonium

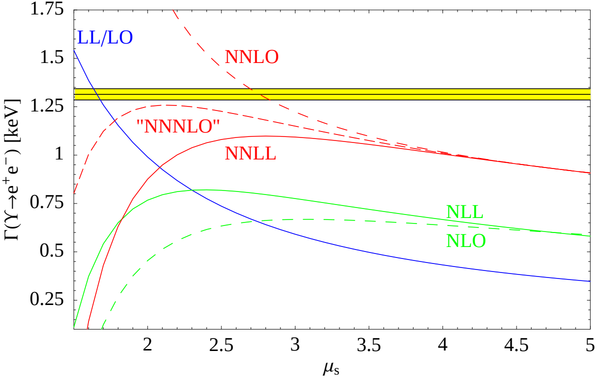

Figure: Prediction for the $\Upsilon(1S)$ decay rate to e^+e^-. We work in the RS' scheme. Pineda, Signer

The effect of the resummation of logarithms is important if compared with just keeping the single logarithm.

NNNLO ?

Coulomb corrections. Penin, Smirnov, Steinhauser; Beneke, Kiyo, Schuller

$$\frac{\delta_3|\phi_1(0)|^2_C}{|\phi_1^{(0)}(0)|^2} \simeq -0.47 \sim \alpha_s^3(\mu) \qquad \text{for } \mu = \mu_B \sim 2\text{GeV}$$

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$$\frac{\delta_3|\phi_1(0)|^2_R}{|\phi_1^{(0)}(0)|^2} \simeq -1.4 \sim \alpha_s^3(\mu) \qquad \text{for } \mu = \mu_B \sim 2\text{GeV}$$

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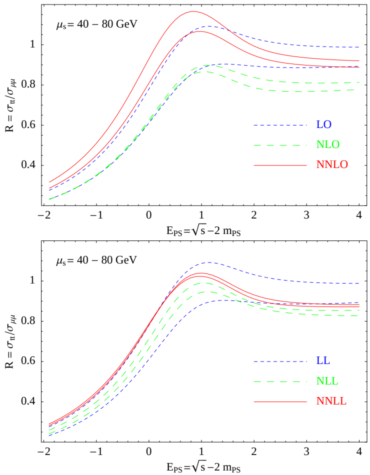

Figure: Threshold scan for $t\bar{t}$. The upper figure shows the fixed order results, LO, NLO and NNLO, whereas the figure belows the RGI results LL, NLL and NNLL are displayed. The soft scale is varied from μ_s=40 GeV to μ_s=80 GeV. Pineda-Signer.

RGI reduces the scale dependence and improves the convergence.

Strong scale dependence ?

There is an strong scale dependence for scales below two GeV (20 for the case of toponium, they seem to have a similar origin) even after the resummation of logarithms.

Possible origin. The scale dependence of the Coulomb corrections.

Possible solution. Solving the Schroedinger equation with the Coulomb equation exactly (numerically). This significantly reduces the scale dependence.

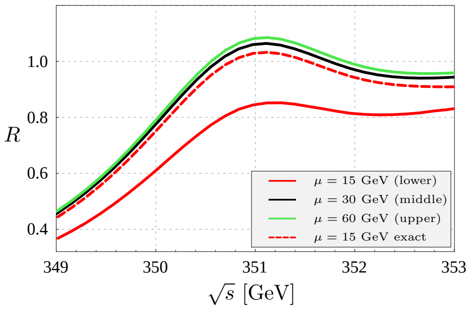

Figure: Top quark pair production cross section (Coulomb corrections only). Scale dependence of the third-order approximation. From hep-ph/0501289.

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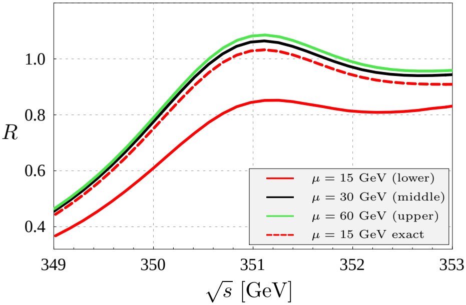

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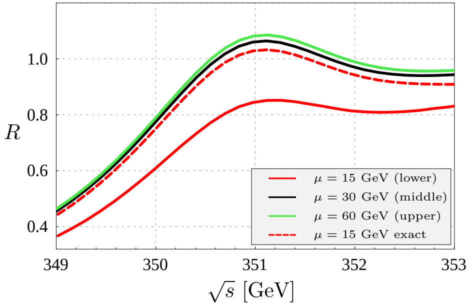

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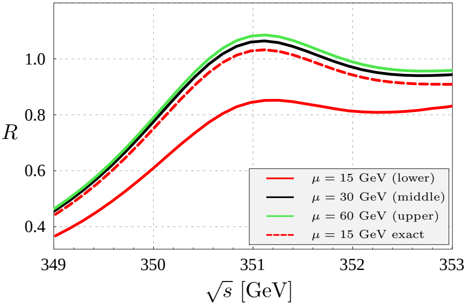

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NON-RELATIVISTIC SUM RULES: BOTTOMONIUM

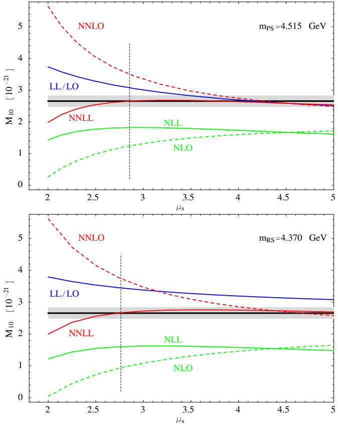

Figure: *The moment M_{10} as a function of μ_s at LO/LL, NLO, NLL, NNLO and NNLL for $m_{b\rm PS}$(2 GeV) = 4.515 GeV in the PS scheme (upper figure), and for $m_{b\rm RS}$(2 GeV) = 4.370 GeV in the RS scheme (lower figure). The experimental moment with its error is also shown (grey band). Pineda, Signer.*

$$m_{b,\rm PS}(2\rm GeV) = 4.52 \pm 0.06 \; GeV,$$

$$m_{b,\rm RS}(2\rm GeV) = 4.37 \pm 0.07 \; GeV.$$

$$\overline{m}_b(\overline{m}_b) = 4.19 \pm 0.06 \, \rm GeV.$$

The perturbative series is sign-alternating. This is the opposite than for electromagnetic decays. The convergence of the perturbative series in sum rules is also better in sum rules than for electromagnetic decays.
NNLO determinations of the bottom sum rules suffer from very huge theoretical uncertainties (which are not always incorporated in the errors): bad scale dependence and bad convergence of the perturbative series. Therefore, they can not provide precise determinations of the bottom mass.

Semi-inclusive radiative decays of $\Upsilon(1S)$

Garcia, Soto:

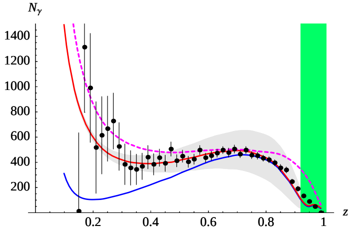

Figure: Photon spectrum from CLEO data. The solid lines are the NLO merging plus the fragmentation contributions: the red and blue line are obtained using different estimates for $\langle \Upsilon(1S)|O_8(^3S_1)|\Upsilon(1S)\rangle$. The grey shaded region is obtained by varying μ_c by $\sqrt{2^{\pm 1}}\mu_c$. The green shaded region shows the zone where the calculation of the shape functions is not reliable. The pink dashed line is the result from Fleming et al., where only color singlet contributions were included.

Brambilla, Garcia, Soto, Vairo:

$$R_\gamma \equiv \frac{Gamma[\Upsilon(1S) \to \gamma X]}{Gamma[\Upsilon(1S) \to X]} \to \alpha_s(M_z) = 0.120^{+0.005}_{-0.006}$$

with claimed accuracy of order $\mathcal{O}(\alpha_s, v^2)$.

CONCLUSIONS

pNRQCD: Effective field theory from QCD that describes Heavy Quarkonium. Main distinction:

- Weak coupling regime (more predictive).
- Strong coupling regime (less predictive).

Smooth connection with potential models → Good description of the gross features of the spectrum. Questions:

- How much of the potential can be understood from perturbation theory.
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- How much of the decays can be understood from perturbation theory.

Battle ground. Weak coupling: behavior of the wave-function at the origin. Understanding inclusive electromagnetic decays and hyperfine splitting of the bottomonium ground state; impact on t-$\bar{t}$ production and sum rules as well. Strong coupling: unquenched simulations for the potentials and connection with $\overline{\mathrm{MS}}$-like schemes.

b-$\bar{b}$ nonrelativistic sum rules and/or $\Upsilon(1S)$ mass → m_b mass.
t-$\bar{t}$ production near threshold → m_t mass.
Semiinclusive radiative decays of $\Upsilon(1S)$ → $\alpha_s(M_z)$.
Radiative transitions, Quark-gluon plasma (Charmonium used as a thermometer), ...

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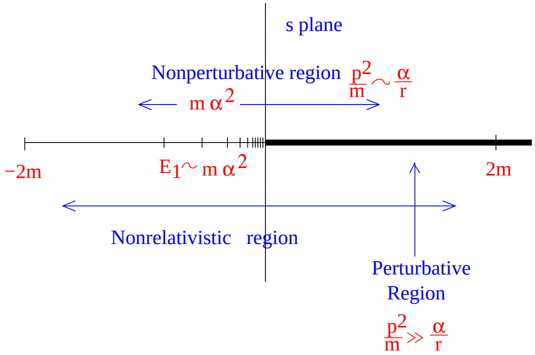
t-$\bar{t}$ production near threshold → m_t mass.
Semiinclusive radiative decays of $\Upsilon(1S) \rightarrow \alpha_s(M_z)$.
Radiative transitions, Quark-gluon plasma (Charmonium used as a thermometer), ...

kinematical situation

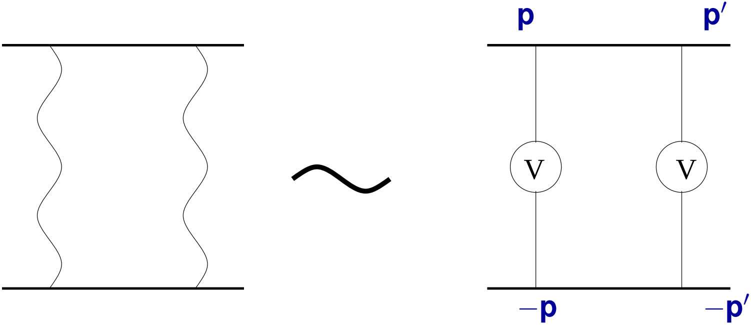


1st approximation: ∞ number of NR (bound states) free particles
Unusual situation in EFTs. What we will get is somewhat unusual from the EFT point of view.

$$\mathcal{L} = \sum_n \psi_n^\dagger(\mathbf{X},t)\left(i\partial_0 + \frac{\boldsymbol{\nabla}_X^2}{2M} - E_n + i\epsilon\right)\psi_n(\mathbf{X},t)$$

$\psi_n(\mathbf{X})$ represents the quark-antiquark bound state

Physical Picture



$$I \sim \int \frac{d^4 q}{(2\pi)^4} V(p,q) \frac{1}{E/2 + q^0 - \mathbf{q}^2/2m + i\epsilon} \frac{1}{E/2 - q^0 - \mathbf{q}^2/2m + i\epsilon} V(q,p')$$

$$V(p,q) \sim \frac{1}{(p-q)^2}$$

Counting (different possibilities):

A) $E \sim mv^2 \quad p^0, p'^0 \sim q^0 \sim |\mathbf{p}| \sim \mathbf{q} \sim \mathbf{p}' \sim mv$

Static propagator:

$$\frac{i}{q^0 + i\epsilon} \to I \sim \delta V(\mathbf{p} - \mathbf{q})$$

B) $E \sim p^0, p'^0 \sim q^0 \sim mv^2 \quad |\mathbf{p}| \sim \mathbf{q} \sim \mathbf{p}' \sim mv$

Nonrelativistic propagator:

$$\frac{i}{q^0 - \mathbf{q}^2/(2m) + i\epsilon}$$

A) Time independent contribution. It gives rise to a correction to the potential

B) Leading contribution:

$$I \sim \int \frac{d^3\mathbf{q}}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}) \frac{1}{E - \mathbf{q}^2/m + i\epsilon} V(\mathbf{q}, \mathbf{p}') \,.$$

This is nothing but the usual NR Quantum Mechanics!!, I can be written as

$$I \sim \langle \mathbf{p} | \hat{V} \frac{1}{E - \mathbf{p}^2/m + i\epsilon} \hat{V} | \mathbf{p}' \rangle$$

This can be done to any order considering ladder loops. Therefore,

$$i\mathcal{A} = -i \langle \mathbf{p} | \left(\hat{V} + \hat{V} \frac{1}{E - \mathbf{p}^2/m + i\epsilon} \hat{V} + \ldots \right) | \mathbf{p}' \rangle \,.$$