# QCD Corrections to inclusive $J/\psi$ production at the B factories

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# Outline

# Introduction

- Color-octet mechanism was proposed based on NRQCD.
- But there are still many difficulties.
  - $J/\psi$ photoproduction at HERA
  - $J/\psi$ production at the B factories
  - $J/\psi$ polarization at the Tevatron
- NLO corrections are important.
  - Data on inelastic $J/\psi$ photoproduction are adequately described by the color singlet channel alone at NLO
  - Double charmonium production at the B factories

# $e^+e^- \to J/\psi + \eta_c$

## Experimantal Data

BELLE: $\sigma[J/\psi + \eta_c] \times B^{\eta_c}[\geq 2] = (25.6 \pm 2.8 \pm 3.4)$ fb
*BARAR*: $\sigma[J/\psi + \eta_c] \times B^{\eta_c}[\geq 2] = (17.6 \pm 2.8^{+1.5}_{-2.1})$ fb
[Abe et al.(2002), Pakhlov(2004), Aubert et al.(2005)]

## LO NRQCD Predictions

$2.3 \sim 5.5$ fb
[Braaten and Lee(2003), Liu et al.(2003), Hagiwara et al.(2003)]

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## NLO QCD corrections

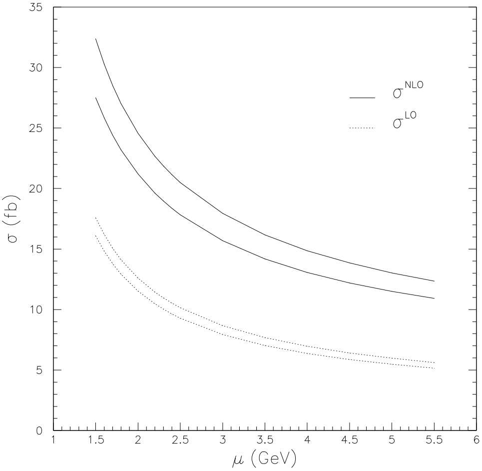
$K \equiv \sigma^{NLO}/\sigma^{LO} \sim 2$

[Zhang et al.(2006), Gong and Wang(2007)]

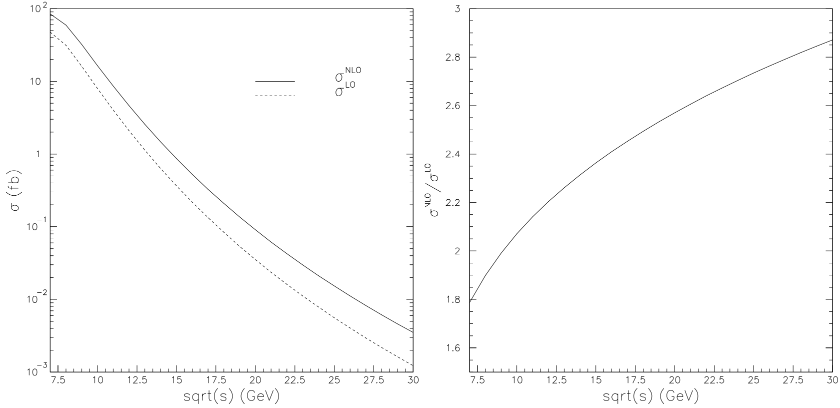
| $m_c$(GeV) | $\mu$ | $\alpha_s(\mu)$ | $\sigma_{LO}$(fb) | $\sigma_{NLO}$(fb) | $\sigma_{NLO}/\sigma_{LO}$ |
|---|---|---|---|---|---|
| 1.5 | $m_c$ | 0.369 | 16.09 | 27.51 | 1.710 |
| 1.5 | $2m_c$ | 0.259 | 7.94 | 15.68 | 1.975 |
| 1.5 | $\sqrt{s}/2$ | 0.211 | 5.27 | 11.14 | 2.114 |
| 1.4 | $m_c$ | 0.386 | 19.28 | 34.92 | 1.811 |
| 1.4 | $2m_c$ | 0.267 | 9.19 | 18.84 | 2.050 |
| 1.4 | $\sqrt{s}/2$ | 0.211 | 5.76 | 12.61 | 2.189 |

Cross sections with different charm quark mass $m_c$ and renormalization scale $\mu$. $\sqrt{s} = 10.6\ \mathrm{GeV}$ is the c.m. energy.

Confirmed the result given by [Zhang et al.(2006)] analytically.




Cross sections as function of the renormalization scale $\mu$ with $|R_s(0)|^2 = 0.978\ \mathrm{GeV}^3$, $\Lambda = 0.338\ \mathrm{GeV}$ and c.m. energy $10.6\ \mathrm{GeV}$. The charm quark mass is chosen as 1.4 GeV (upper curves) and 1.5 GeV (lower curves).




Cross sections and K factor as function of the center-of-mass energy with $|R_s(0)|^2 = 0.978\ \mathrm{GeV}^3$ and $\Lambda = 0.338\ \mathrm{GeV}$. Renormalization scale $\mu$ is set at half of the center-of-mass energy and $m_c = 1.5\ \mathrm{GeV}$.

# $e^+e^- \longrightarrow J/\psi + J/\psi$

## Problem

LO NRQCD prediction indicates that the cross section of this process is large than that of $J/\psi + \eta_c$ production by a factor of 1.8, but no evidence for this process was found at the B factories.
[Bodwin et al.(2003a), Abe et al.(2004)]

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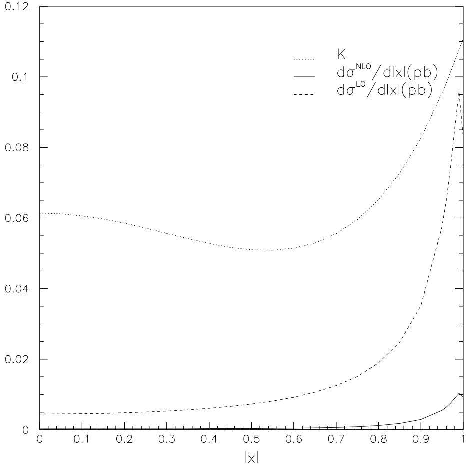
## NLO QCD corrections

- Greatly decreased, with a K factor ranging from $-0.31 \sim 0.25$ depending on the renormalization scale.
- Might explain the situation.

[Gong and Wang(2008b)]

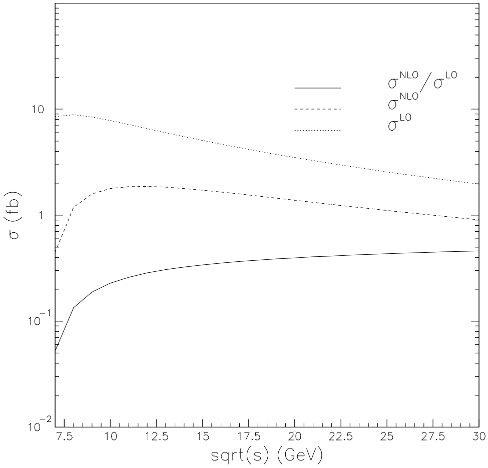
| $m_c$(GeV) | $\mu$ | $\alpha_s(\mu)$ | $\sigma_{LO}$(fb) | $\sigma_{NLO}$(fb) | $\sigma_{NLO}/\sigma_{LO}$ |
|---|---|---|---|---|---|
| 1.5 | $m_c$ | 0.369 | 7.409 | -2.327 | -0.314 |
| 1.5 | $2m_c$ | 0.259 | 7.409 | 0.570 | 0.077 |
| 1.5 | $\sqrt{s}/2$ | 0.211 | 7.409 | 1.836 | 0.248 |
| 1.4 | $m_c$ | 0.386 | 9.137 | -3.350 | -0.367 |
| 1.4 | $2m_c$ | 0.267 | 9.137 | 0.517 | 0.057 |
| 1.4 | $\sqrt{s}/2$ | 0.211 | 9.137 | 2.312 | 0.253 |

Cross sections with different charm quark mass $m_c$ and renormalization scale $\mu$, and $\sqrt{s} = 10.6$ GeV.




Differential cross section as function of $|x|$ where $x = \cos(\theta)$.

$\theta$ is the angle between the $J/\psi$ and the beam, and $K = \frac{\mathrm{d}\sigma^{\mathrm{NLO}}}{\mathrm{d}|x|} / \frac{\mathrm{d}\sigma^{\mathrm{LO}}}{\mathrm{d}|x|}$ is the ratio of differential cross section of NLO to LO. $m_c$ is set as 1.5 GeV and $\mu = \sqrt{s}$ is taken.




Cross section as function of the c.m. energy $\sqrt{s}$ with $m_c = 1.5$ GeV and $\mu = \sqrt{s}/2$.

## LO NRQCD Predictions:

| | |
|---|---|
| $e^+e^- \to J/\psi + c\bar{c}$ | $0.07 \sim 0.20$pb |
| $e^+e^- \to J/\psi + gg$ | $0.15 \sim 0.3$pb |
| $e^+e^- \to J/\psi^{(8)}({}^3P_J, {}^1S_0) + g$ | $0.3 \sim 0.8$pb |

## Experimental Data:

| | |
|---|---|
| *BARAR* | $\sigma[e^+e^- \to J/\psi + X] = (2.54 \pm 0.21 \pm 0.21)$ pb |
| CLEO | $\sigma[e^+e^- \to J/\psi + X] = (1.9 \pm 0.20)$ pb |
| BELLE | $\sigma[e^+e^- \to J/\psi + X] = (1.45 \pm 0.10 \pm 0.13)$ pb |
| | $\sigma[e^+e^- \to J/\psi + c\bar{c} + X] = (0.87^{+0.21}_{-0.19} \pm 0.17)$ pb |

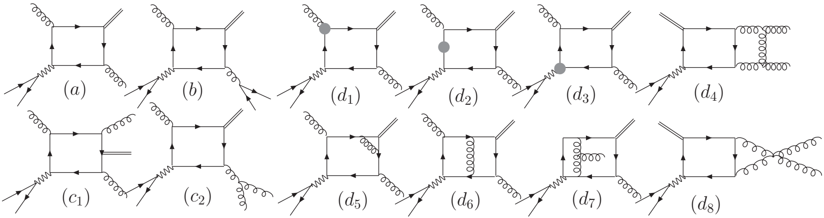
[Aubert et al.(2001), Aubert et al.(2005), Briere et al.(2004), Abe et al.(2002a), Abe et al.(2002)]

## New BELLE Data

$$\begin{aligned}
\sigma[e^+e^- \to J/\psi + X] &= (1.17 \pm 0.02 \pm 0.07)\ \mathrm{pb} \\
\sigma[e^+e^- \to J/\psi + c\bar{c}] &= (0.74 \pm 0.08^{+0.09}_{-0.08})\ \mathrm{pb} \\
\sigma[e^+e^- \to J/\psi + X_{\mathrm{non}-c\bar{c}}] &= (0.43 \pm 0.09 \pm 0.09)\ \mathrm{pb}
\end{aligned}$$

[Pakhlov et al.(2009)]

# $e^+e^- \to J/\psi + gg$




Typical Feynman Diagrams. (a): LO; (b) $e^+e^- \to J/\psi + gq\bar{q}$; (c) $e^+e^- \to J/\psi + ggg$; (d): One-loop. Groups $(d_1) - (d_3)$ are the counter-term diagrams, including corresponding loop diagrams.

# LO Result

$$\begin{aligned}\sigma^{(0)} &= -\frac{\alpha^2\alpha_s^2 e_c^2|R_s(0)|^2}{9m_c^5\hat{s}^3(\hat{s}-1)}\left\{18(\hat{s}^2-2\hat{s}+2)+\frac{2\hat{s}(5\hat{s}^2-14\hat{s}+3)}{\hat{s}-1}\ln(\hat{s})\right.\\ &\quad+\frac{2(4\hat{s}^2-9\hat{s}+8)\beta}{\hat{s}-1}\ln(2\hat{s}-1-2\beta)+\frac{2\hat{s}^3-\hat{s}^2-12\hat{s}+8}{\hat{s}-1}\\ &\quad\left.\times\left[\mathrm{Li}_2\left(\frac{2(\hat{s}-1)}{\hat{s}+\beta-1}\right)+\mathrm{Li}_2\left(\frac{2(\hat{s}-1)}{\hat{s}-\beta-1}\right)\right]\right\}+\mathcal{O}(\epsilon),\end{aligned}$$

$$\hat{s}=s/(4m_c^2) \quad \beta=\sqrt{\hat{s}(\hat{s}-1)}$$

# Renormalization Constants

$$
\begin{aligned}
\delta Z_m^{\rm OS} &= -3C_F\frac{\alpha_s}{4\pi}\left[\frac{1}{\epsilon_{UV}} - \gamma_E + \ln\frac{4\pi\mu^2}{m_c^2} + \frac{4}{3}\right], \\
\delta Z_2^{\rm OS} &= -C_F\frac{\alpha_s}{4\pi}\left[\frac{1}{\epsilon_{UV}} + \frac{2}{\epsilon_{IR}} - 3\gamma_E + 3\ln\frac{4\pi\mu^2}{m_c^2} + 4\right], \\
\delta Z_3^{\rm OS} &= \frac{\alpha_s}{4\pi}\left[(\beta_0' - 2C_A)\left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}}\right)\right. \\
&\quad \left. -\frac{4}{3}T_F\left(\frac{1}{\epsilon_{UV}} - \gamma_E + \ln\frac{4\pi\mu^2}{m_c^2}\right)\right], \\
\delta Z_g^{\overline{\rm MS}} &= -\frac{\beta_0}{2}\frac{\alpha_s}{4\pi}\left[\frac{1}{\epsilon_{UV}} - \gamma_E + \ln(4\pi)\right].
\end{aligned}
$$

### Real correction splitting

$$\sigma^R = \sigma^S + \sigma^{HC} + \sigma^{H\overline{C}}$$

### Soft region

$$d\sigma^S = d\sigma^{(0)} \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu^2}{s}\right)^{\epsilon} \left(\frac{A_2^S}{\epsilon^2} + \frac{A_1^S}{\epsilon} + A_0^S\right)$$

$$A_2^S = 6, \quad A_1^S = -12\ln\delta_s - 6\ln(\sin^2\frac{\theta_g}{2}), \quad A_0^S = \frac{(A_1^S)^2}{12} + 6\mathrm{Li}_2(\cos^2\frac{\theta_g}{2})$$

### Hard Collinear Region

$$\begin{aligned}
d\sigma^{HC} &= d\sigma^{(0)} \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu^2}{s}\right)^{\epsilon} \left(\frac{A_1^{HC}}{\epsilon} + A_0^{HC}\right), \\
A_1^{HC} &= 11 + 6\ln\delta_s^{(4)} + 6\ln\delta_s^{(5)} - \frac{2}{3}n_{lf}, \\
A_0^{HC} &= \frac{67}{3} - \frac{10}{9}n_{lf} - 2\pi^2, -3\ln^2\delta_s^{(4)} - 3\ln^2\delta_s^{(5)} - \ln\delta_c A_1^{HC}, \\
\delta_s^{(j)} &= \delta_s/[1-(p_3+p_j)^2/s]
\end{aligned}$$

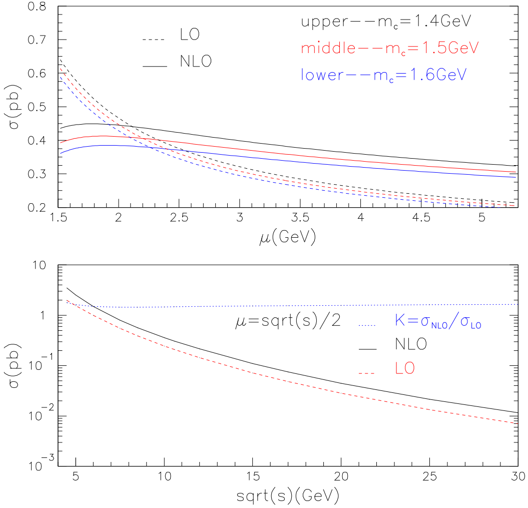
## Cross section at NLO

$$\sigma^{(1)} = \sigma^{(0)} \left\{ 1 + \frac{\alpha_s(\mu)}{\pi} \left[ a(\hat{s}) + \beta_0 \ln\left(\frac{\mu}{2m_c}\right) \right] \right\}$$

| $m_c$(GeV) | $\alpha_s(\mu)$ | $\sigma^{(0)}$(pb) | $a(\hat{s})$ | $\sigma^{(1)}$(pb) | $\sigma^{(1)}/\sigma^{(0)}$ |
|---|---|---|---|---|---|
| 1.4 | 0.267 | 0.341 | 2.35 | 0.409 | 1.20 |
| 1.5 | 0.259 | 0.308 | 2.57 | 0.373 | 1.21 |
| 1.6 | 0.252 | 0.279 | 2.89 | 0.344 | 1.23 |

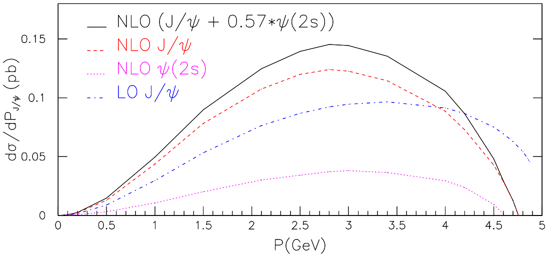
Cross sections with different charm quark mass $m_c$ where the renormalization scale $\mu = 2m_c$ and $\sqrt{s} = 10.6$ GeV.

Consistent with another individual calculation by [Ma et al.(2009)].

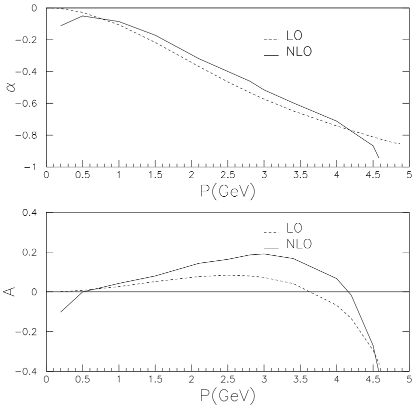



Cross sections as function of the renormalization scale $\mu$ and the center-of-mass energy of $e^+e^-$ $\sqrt{s}$.

# Momentum distribution




Momentum distribution of $J/\psi$ production with $m_c = 1.5$ GeV and $\mu = 2m_c$.




Polarization parameter $\alpha$ and angular distribution parameter $A$

of $J/\psi$ as functions of $p$ with $m_c = 1.5$ GeV and $\mu = 2m_c$.

$$\frac{d^2\sigma}{d\cos\theta\, dp} = S(p)[1 + A(p)\cos\theta]$$

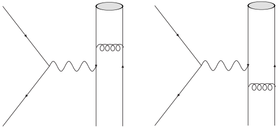
$$\alpha = \frac{\sigma_T - 2\sigma_L}{\sigma_T + 2\sigma_L}$$

Results on the left contain potentially large numerical errors in our calculation for $p < 0.5$ GeV or $p > 4.2$ GeV due to the cancellation of large numbers.

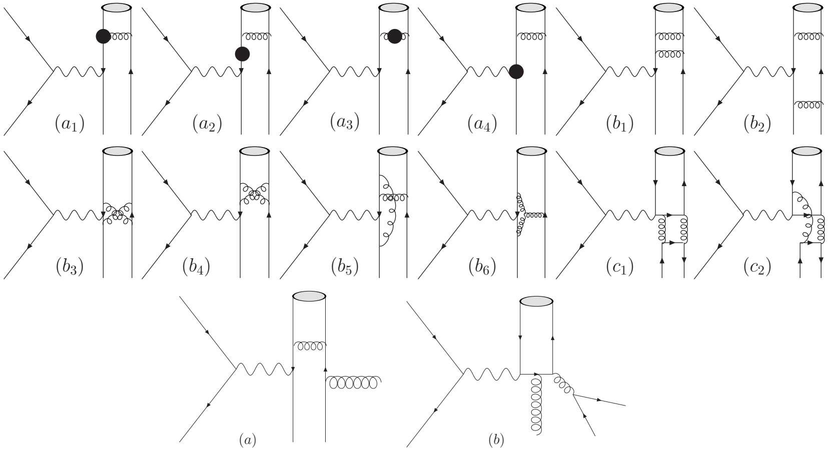
# $e^+e^- \to J/\psi + c\bar{c}$

$$
\begin{aligned}
\frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}E_{J/\psi}} = \; & \frac{4\alpha^2\alpha_s^2 e_c^2 |R_s(0)|^2}{27 m_c^6 \hat{s}^{5/2}} \Bigg\{ \Bigg[ -\frac{128(2\hat{s}+1)(\hat{s}-1)^2\hat{s}^2}{(\hat{s}_{c\bar{c}}-1+\hat{s})^6} + \frac{32(6\hat{s}-1)(2\hat{s}+1)(\hat{s}-1)\hat{s}}{(\hat{s}_{c\bar{c}}-1+\hat{s})^5} - \frac{8(8\hat{s}-1)(2\hat{s}+1)\hat{s}}{(\hat{s}_{c\bar{c}}-1+\hat{s})^4} \\
& + \frac{4(8\hat{s}^3+12\hat{s}^2+3)}{\hat{s}(\hat{s}_{c\bar{c}}-1+\hat{s})^3} - \frac{52\hat{s}^4-30\hat{s}^3-42\hat{s}^2-\hat{s}+9}{(\hat{s}-1)\hat{s}^2(\hat{s}_{c\bar{c}}-1+\hat{s})^2} + \frac{2(10\hat{s}^5-18\hat{s}^4+18\hat{s}^3-5\hat{s}^2-2\hat{s}+3)}{(\hat{s}-1)^2\hat{s}^3(\hat{s}_{c\bar{c}}-1+\hat{s})} \\
& - \frac{2(4\hat{s}+3)}{\hat{s}^3(\hat{s}_{c\bar{c}}-1-\hat{s})} + \frac{(2\hat{s}+3)(2\hat{s}+1)}{\hat{s}^2(\hat{s}_{c\bar{c}}-1-\hat{s})^2} - \frac{4(\hat{s}+2)}{(\hat{s}-1)^2\hat{s}_{c\bar{c}}} \Bigg] \Delta_1\Delta_2\Delta_3 + \Bigg[ -\frac{2(8\hat{s}^3+2\hat{s}^2+10\hat{s}-3)}{\hat{s}(\hat{s}_{c\bar{c}}-1+\hat{s})^2} \\
& + \frac{6(6\hat{s}^3-3\hat{s}^2+1)}{\hat{s}^2(\hat{s}_{c\bar{c}}-1+\hat{s})} + \frac{(2\hat{s}^3+11\hat{s}^2-6)}{\hat{s}^2(\hat{s}_{c\bar{c}}-1-\hat{s})} - \frac{2(4\hat{s}^2-3)}{\hat{s}(\hat{s}_{c\bar{c}}-1-\hat{s})^2} - \frac{2(2\hat{s}+3)(2\hat{s}+1)}{(\hat{s}_{c\bar{c}}-1-\hat{s})^3} - 16 \Bigg] \\
& \times \ln\left( \frac{\hat{s}+1-\hat{s}_{c\bar{c}}+\Delta_1\Delta_3/\Delta_2}{\hat{s}+1-\hat{s}_{c\bar{c}}-\Delta_1\Delta_3/\Delta_2} \right) \Bigg\} + \mathcal{O}(\epsilon)
\end{aligned}
$$

$$
\hat{s} = \frac{s}{4m_c^2}, \qquad \hat{s}_{c\bar{c}} = \frac{(p_4+p_5)^2}{4m_c^2},
$$
$$
\Delta_1 = \sqrt{\hat{s}_{c\bar{c}}-1}, \qquad \Delta_2 = \sqrt{\hat{s}_{c\bar{c}}},
$$
$$
\Delta_3 = \lambda^{1/2}(\hat{s}, \hat{s}_{c\bar{c}}, 1) \equiv \sqrt{(\hat{s}-\hat{s}_{c\bar{c}}-1)^2 - 4\hat{s}_{c\bar{c}}}.
$$



# Typical Feynman Didgrams at NLO



- Real correction splitting: $\sigma^R = \sigma^S + \sigma^{H\overline{C}}$ no hard collinear part this time
- soft region

$$d\sigma^S = d\sigma^{(0)} \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu^2}{s}\right)^{\epsilon} \left(\frac{A_1}{\epsilon} + A_0\right).$$

- in the usual $e^+e^-$ rest frame

$$A_1 = 2C_F \left[1 - \frac{k_\beta}{2\sqrt{\Delta}} \ln \frac{k_\beta + \sqrt{\Delta}}{k_\beta - \sqrt{\Delta}}\right],$$

$$A_0 = 2C_F \left[ -2\ln\delta_s + \frac{1}{2\beta_c} \ln \frac{1+\beta_c}{1-\beta_c} + \frac{1}{2\beta_{\bar{c}}} \ln \frac{1+\beta_{\bar{c}}}{1-\beta_{\bar{c}}} + \frac{k_\beta}{\sqrt{\Delta}} \ln\delta_s \ln \frac{k_\beta + \sqrt{\Delta}}{k_\beta - \sqrt{\Delta}} - I(\beta_c, \beta_{\bar{c}}, \cos\theta_{c\bar{c}})\right],$$

$$k_\beta = 1 - \beta_c\beta_{\bar{c}} \cos\theta_{c\bar{c}}, \qquad I(\beta_c, \beta_{\bar{c}}, \cos\theta) = \int_0^1 dx \frac{1}{f(x)[1-f(x)^2]} \ln \frac{1+f(x)}{1-f(x)},$$

$$\Delta = k_\beta^2 - (1-\beta_c^2)(1-\beta_{\bar{c}}^2), \quad f(x) = \left[(1-x)^2\beta_c^2 + 2x(1-x)\beta_c\beta_{\bar{c}}\cos\theta + x^2\beta_{\bar{c}}^2\right]^{1/2}.$$

- Real correction splitting: $\sigma^R = \sigma^S + \sigma^{H\overline{C}}$ no hard collinear part this time
- soft region

$$d\sigma^S = d\sigma^{(0)} \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu^2}{s}\right)^\epsilon \left(\frac{A_1}{\epsilon} + A_0\right).$$

- in the usual $e^+e^-$ rest frame

$$A_1 = 2C_F\left[1 - \frac{k_\beta}{2\sqrt{\Delta}} \ln \frac{k_\beta + \sqrt{\Delta}}{k_\beta - \sqrt{\Delta}}\right],$$

$$A_0 = 2C_F\left[-2\ln\delta_s + \frac{1}{2\beta_c}\ln\frac{1+\beta_c}{1-\beta_c} + \frac{1}{2\beta_{\bar{c}}}\ln\frac{1+\beta_{\bar{c}}}{1-\beta_{\bar{c}}} + \frac{k_\beta}{\sqrt{\Delta}}\ln\delta_s \ln\frac{k_\beta + \sqrt{\Delta}}{k_\beta - \sqrt{\Delta}} - I(\beta_c, \beta_{\bar{c}}, \cos\theta_{c\bar{c}})\right],$$

$$k_\beta = 1 - \beta_c\beta_{\bar{c}}\cos\theta_{c\bar{c}}, \qquad I(\beta_c, \beta_{\bar{c}}, \cos\theta) = \int_0^1 dx \frac{1}{f(x)[1-f(x)^2]} \ln\frac{1+f(x)}{1-f(x)},$$

$$\Delta = k_\beta^2 - (1-\beta_c^2)(1-\beta_{\bar{c}}^2), \quad f(x) = \left[(1-x)^2\beta_c^2 + 2x(1-x)\beta_c\beta_{\bar{c}}\cos\theta + x^2\beta_{\bar{c}}^2\right]^{1/2}.$$

- in the $c\bar{c}$ rest frame

$$A_1 = 2C_F\left[1 - \frac{1+\beta^2}{2\beta}\ln\frac{1+\beta}{1-\beta}\right],$$

$$A_0 = 2C_F\left[-2\ln\delta_s + \frac{1}{\beta}\ln\frac{1+\beta}{1-\beta} - \frac{1+\beta^2}{\beta}\left(\mathrm{Li}_2\frac{2\beta}{1+\beta} + \frac{1}{4}\ln^2\frac{1+\beta}{1-\beta} - \ln\delta_s\ln\frac{1+\beta}{1-\beta}\right)\right],$$

$$\beta = \frac{|p_4|}{E_4} = \frac{|p_5|}{E_5} = \sqrt{1 - \frac{1}{\hat{s}_{c\bar{c}}}} = \frac{\Delta_1}{\Delta_2}.$$

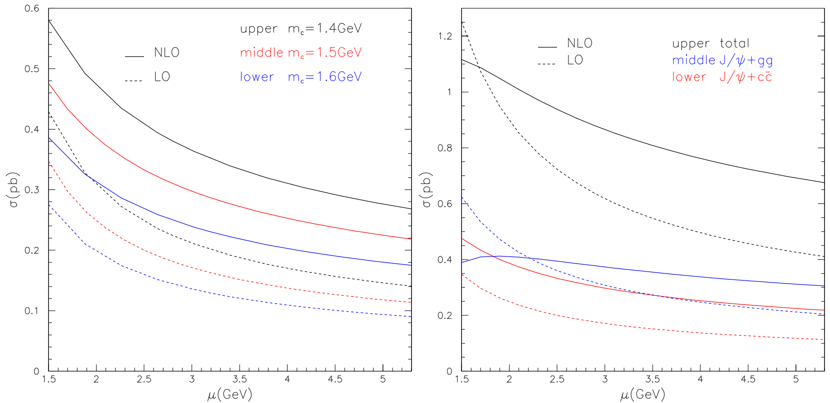
## Cross section at NLO

$$\sigma^{(1)} = \sigma^{(0)} \left\{ 1 + \frac{\alpha_s(\mu)}{\pi} \left[ a(\hat{s}) + \beta_0 \ln\left(\frac{\mu}{2m_c}\right)\right]\right\}$$

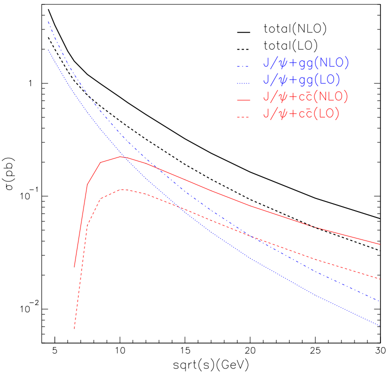
| $m_c$(GeV) | $\alpha_s(\mu)$ | $\sigma^{(0)}$(pb) | $a(\hat{s})$ | $\sigma^{(1)}$(pb) | $\sigma^{(1)}/\sigma^{(0)}$ |
|---|---|---|---|---|---|
| 1.4 | 0.267 | 0.224 | 8.19 | 0.380 | 1.70 |
| 1.5 | 0.259 | 0.171 | 8.94 | 0.298 | 1.74 |
| 1.6 | 0.252 | 0.129 | 9.74 | 0.230 | 1.78 |

Cross sections with different charm quark mass $m_c$ with the renormalization scale $\mu = 2m_c$ and $\sqrt{s} = 10.6$ GeV.

Consistent with the former result given by [Zhang and Chao(2007)].



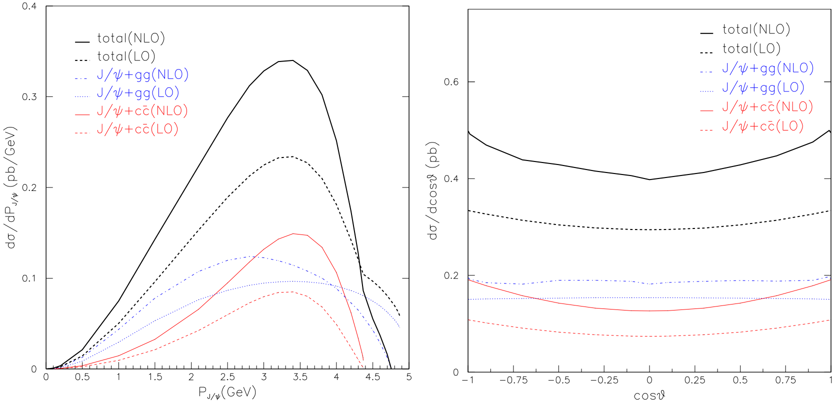

Cross sections for $J/\psi c\bar{c}$, $J/\psi gg$ and total, as function of the renormalization scale $\mu$.



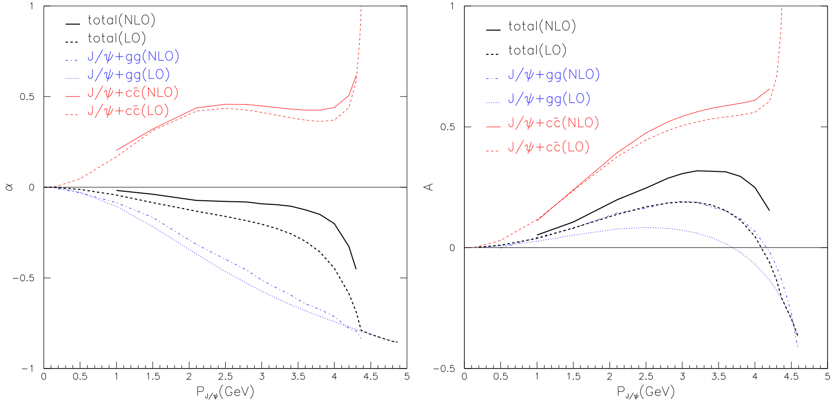

Cross sections for $J/\psi c\bar{c}$, $J/\psi gg$ and total, as function of the center-of-mass energy of $e^+e^-$ $\sqrt{s}$ with the renormalization scale $\mu = \sqrt{s}/2$ (left figure) and $\mu = 2m_c$ (right figure).

$$\sigma^{(0)}_{c\bar{c}} = \frac{8\alpha^2\alpha_s^2 e_c^2|R_s(0)|^2}{27m_c^5} \times \frac{59\pi}{1024\sqrt{2}}\xi^3_{c\bar{c}} + \mathcal{O}(\xi^4_{c\bar{c}}), \quad \xi_{c\bar{c}} = \sqrt{\hat{s}} - 2$$

$$\sigma^{(0)}_{gg} = \frac{\alpha^2\alpha_s^2 e_c^2|R_s(0)|^2}{9m_c^5} \times \frac{8}{3}\xi_{gg} + \mathcal{O}(\xi^2_{gg}), \quad \xi_{gg} = \sqrt{\hat{s}} - 1$$




Momentum and angular distribution of $J/\psi$ production.



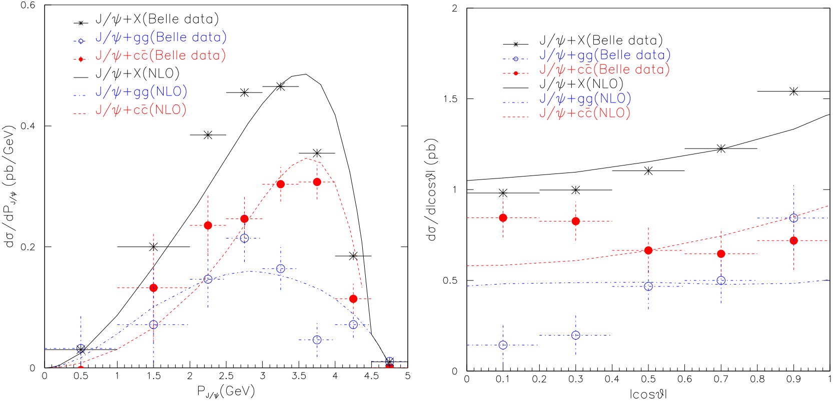

Polarization parameter $\alpha$ and angular distribution parameter $A$ of $J/\psi$ as functions of $p$.

Use Brodsky, Lepage and Mackenzie (BLM) scale setting [Brodsky et al.(1983)]

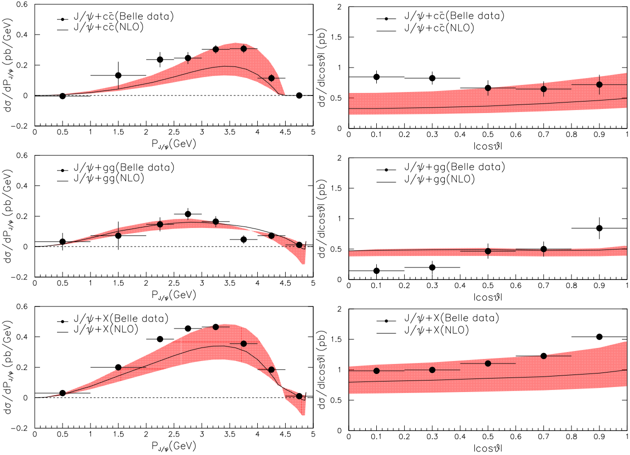
$$\sigma^{(1)} = \sigma^{(0)}(\mu^*)[1 + \frac{\alpha_s(\mu^*)}{\pi} b(\hat{s})].$$

| $m_c$(GeV) | $\alpha_s(\mu^*)$ | $\sigma^{(0)}$(pb) | $b(\hat{s})$ | $\sigma^{(1)}$(pb) | $\sigma^{(1)}/\sigma^{(0)}$ | $\mu^*$(GeV) |
|---|---|---|---|---|---|---|
| 1.4 | 0.348 | 0.381 | 3.77 | 0.540 | 1.42 | 1.65 |
| 1.5 | 0.339 | 0.293 | 4.31 | 0.429 | 1.47 | 1.72 |
| 1.6 | 0.332 | 0.222 | 4.90 | 0.337 | 1.52 | 1.79 |

Cross sections with different charm quark mass $m_c$. The renormalization scale $\mu = \mu^* \sim m_c$.




Momentum distribution of inclusive $J/\psi$ production with $\mu = \mu^*$ and $m_c = 1.4$ GeV is taken for the $J/\psi cc$ channel. The contribution from the feed-down of $\psi'$ has been added to all curves by multiplying a factor of 1.29.




Momentum and angular distributions of inclusive $J/\psi$ production.

The contribution from the feed-down of $\psi'$ has been added to all curves by multiplying a factor of 1.29.

- NLO QCD corrections to both the $J/\psi c\bar{c}$ and $J/\psi gg$ channels have been calculated.
- Very good convergence behaviour is found in the $J/\psi gg$ channel, with a K factor of about 1.20 and significantly improved scale dependence. And the prediction for the total cross section fits the data well.
- A large K factor (about 1.70) is obtained in the $J/\psi c\bar{c}$ channel, but the QCD perturbative expansion can be improved if the BLM scale setting is adopted. And with that scale, the results can account for the new data.
- The momentum distribution of both channels are consistent with data.
- The angular distribution of neither channel can fit the data, unless they are added together.
- Further experiment measurement on the $J/\psi$ polarization is expected.

Thank you!

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