Finite Size Scaling and Universality in SU(2) at Finite Temperature

Yuzhi Liu

yuzhi-liu@uiowa.edu

Department of Physics and Astronomy

The University of Iowa

with A. Denbleyker, Y. Meurice and A. Velytsky

THE UNIVERSITY OF IOWA

Outline

- Introduction
- The Model
- Details
- Conclusion

Introduction

- Finite Size Scaling (FSS)
- Universality
- Binder Cumulant

Finite Size Scaling

- FSS: Extract infinite volume system information from finite volume.
- The Idea: the linear dimension L scales with the correlation length ξ. $1/L$ can be treated as a relevant variable.
  $\xi \propto t^{1/\nu}$
- Linearization
- Binder K 1981 *Z. Phys.* **B43** 119–140
  Binder K and Luijten E 2001 *Phys. Rept.* **344** 179–253

THE UNIVERSITY OF IOWA

Universality

- Universality: No dependence on any microscopic details of the system.
  Dependence on the internal symmetry and the number of the space dimension.
- Critical Exponent: α, β, γ, ν, ω, $\cdots$
- $(3+1)-d\ SU(2)$ guage theory $\longleftrightarrow 3-d$ Ising model.
  Benjamin Svetitsky and Laurence G. Yaffe 1982 *Nucl. Phys.*, B210:423.

THE UNIVERSITY OF IOWA

Binder Cumulant

- Binder Cumumant:

$$g_4 = 1 - \frac{\langle P^4 \rangle}{3\langle P^2 \rangle^2}, \quad P = \frac{1}{N_\sigma^3} \sum_{\vec{x}} \frac{1}{2} \mathrm{Tr} \prod_{\tau=1}^{N_\tau} U_{\tau,\vec{x};0}$$

THE UNIVERSITY OF IOWA

Binder Cumulant

- Binder Cumumant:

$$g_4 = 1 - \frac{\langle P^4 \rangle}{3\langle P^2 \rangle^2}, \qquad P = \frac{1}{N_\sigma^3} \sum_{\vec{x}} \frac{1}{2} \mathrm{Tr} \prod_{\tau=1}^{N_\tau} U_{\tau,\vec{x};0}$$

Simplified notation:

$$B_4 = \frac{\langle P^4 \rangle}{3\langle P^2 \rangle^2}$$

THE UNIVERSITY OF IOWA

Binder Cumulant

- Binder Cumumant:

$$g_4 = 1 - \frac{\langle P^4 \rangle}{3\langle P^2 \rangle^2}, \quad P = \frac{1}{N_\sigma^3} \sum_{\vec{x}} \frac{1}{2} \mathrm{Tr} \prod_{\tau=1}^{N_\tau} U_{\tau,\vec{x};0}$$

"Renormalized Coupling Constant":

$$-3g_4$$

(depreciated by Kurt Binder).

The Model

- We work on SU(2) gauge theory in 3+1 dimensions. $N_\tau \times N_\sigma^3$. $N_\tau = 4, N_\sigma = 4, 6, 8, 10, 12, 16, \cdots$
- The Assumption:

$$g_4 = g_4(u_\kappa N_\sigma^{y_\kappa}, u_1 N_\sigma^{-\omega}, \dots)$$

u_κ: the only relevant scaling variable.
u_i: the irrelevant scaling variable.

$$\begin{aligned} u_\kappa &= \kappa + u_\kappa^{(2)} \kappa^2 + \dots \\ u_1 &= u_1^{(0)} + u_1^{(1)} \kappa + \dots \end{aligned}$$

THE UNIVERSITY OF IOWA

The Model

- Under RG Transformation:

  $a \longrightarrow \ell a$

  $N_\sigma \longrightarrow N_\sigma/\ell$

  $u_\kappa \longrightarrow \ell^{1/\nu} u_\kappa$

  $u_1 \longrightarrow \ell^{-\omega} u_1$

- The Main Assumption:

$$g_4(\beta, N_\sigma) = g_4(\beta_c, \infty) + f_1 \kappa N_\sigma^{1/\nu} + f_2 \kappa^2 N_\sigma^{2/\nu} + (c_0 + c_1 \kappa N_\sigma^{1/\nu}) N_\sigma^{-\omega}$$

where

$$\kappa = (\beta - \beta_c)/\beta_c$$

THE UNIVERSITY OF IOWA

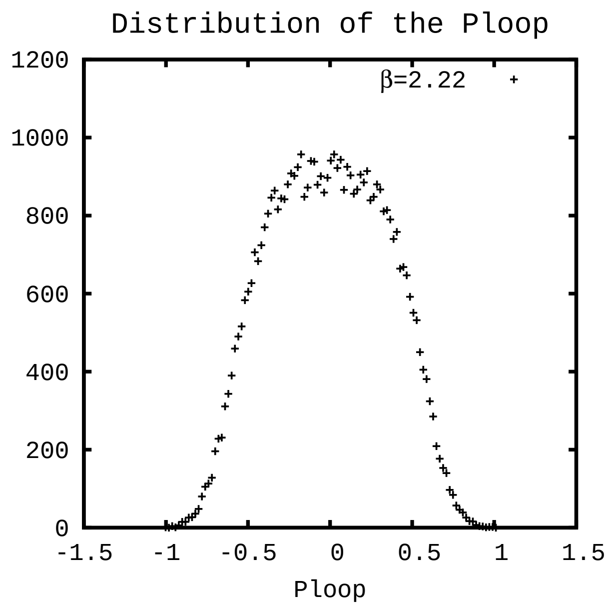
The Model

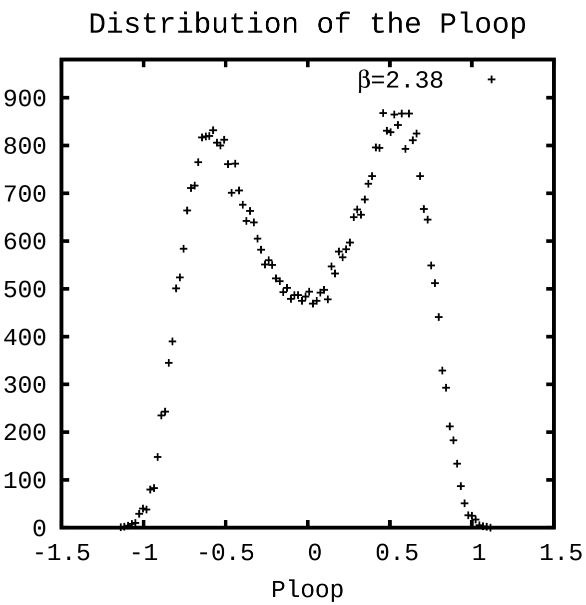
- Spin Model: $\beta \sim 1/T$
- Lattice Gauge Model: $\beta = 2N_c/g^2$, $T = 1/(N_\tau a)$
  Under one-loop scaling:
  $(T - T_c)/T_c \simeq (\beta - \beta_c)12\pi^2/11N_c^2$.

Details

- Properties of the Data
- Determination of FSS Region
- Critical Temperature
- Error Analysis
- Critical Exponent

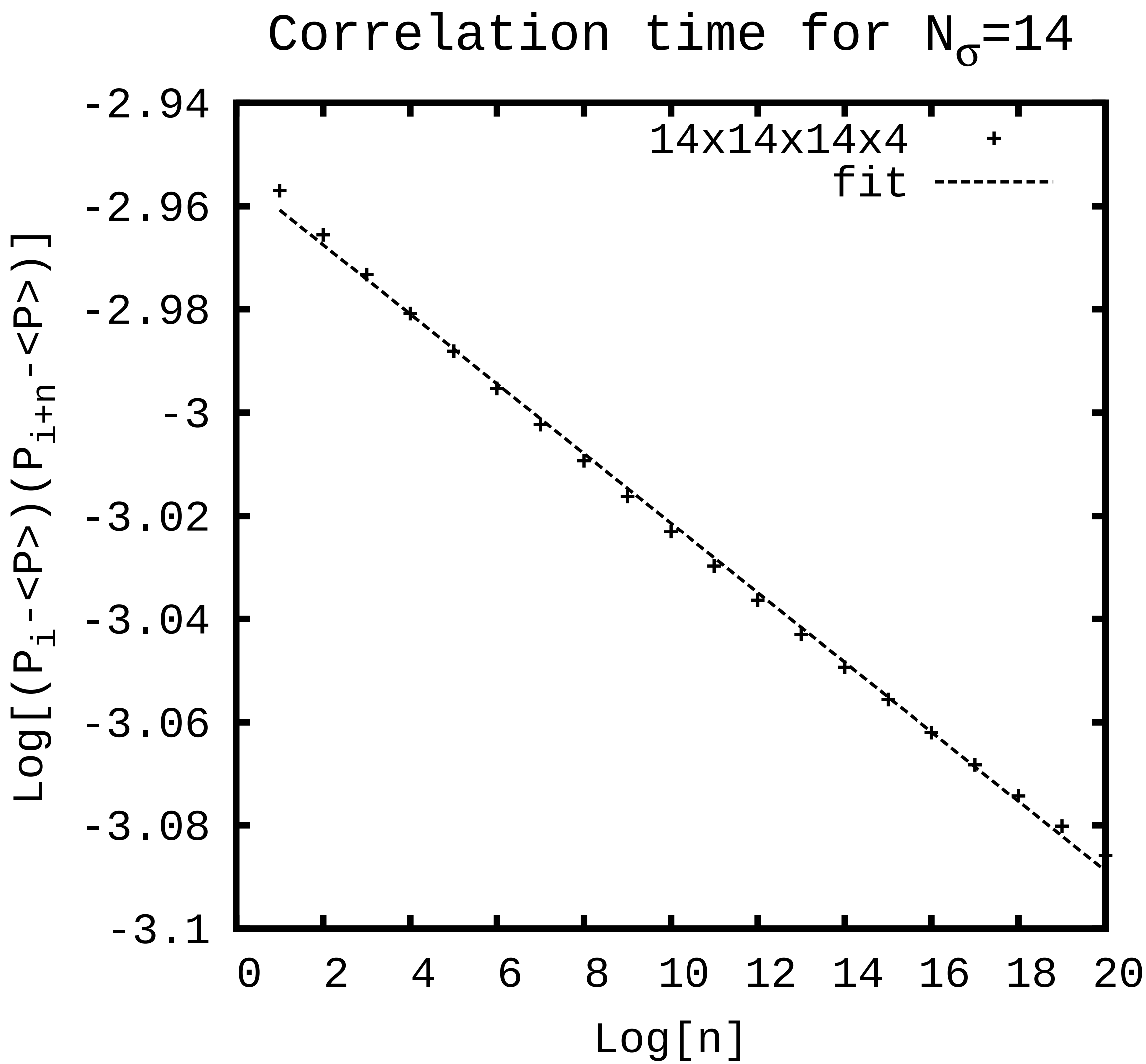
Properties of the Data





THE UNIVERSITY OF IOWA

Properties of the Data



THE UNIVERSITY OF IOWA

Determination of FSS Region

- FSS Region: $|\kappa| N^{1/\nu} \sim 1$
- Our estimate:

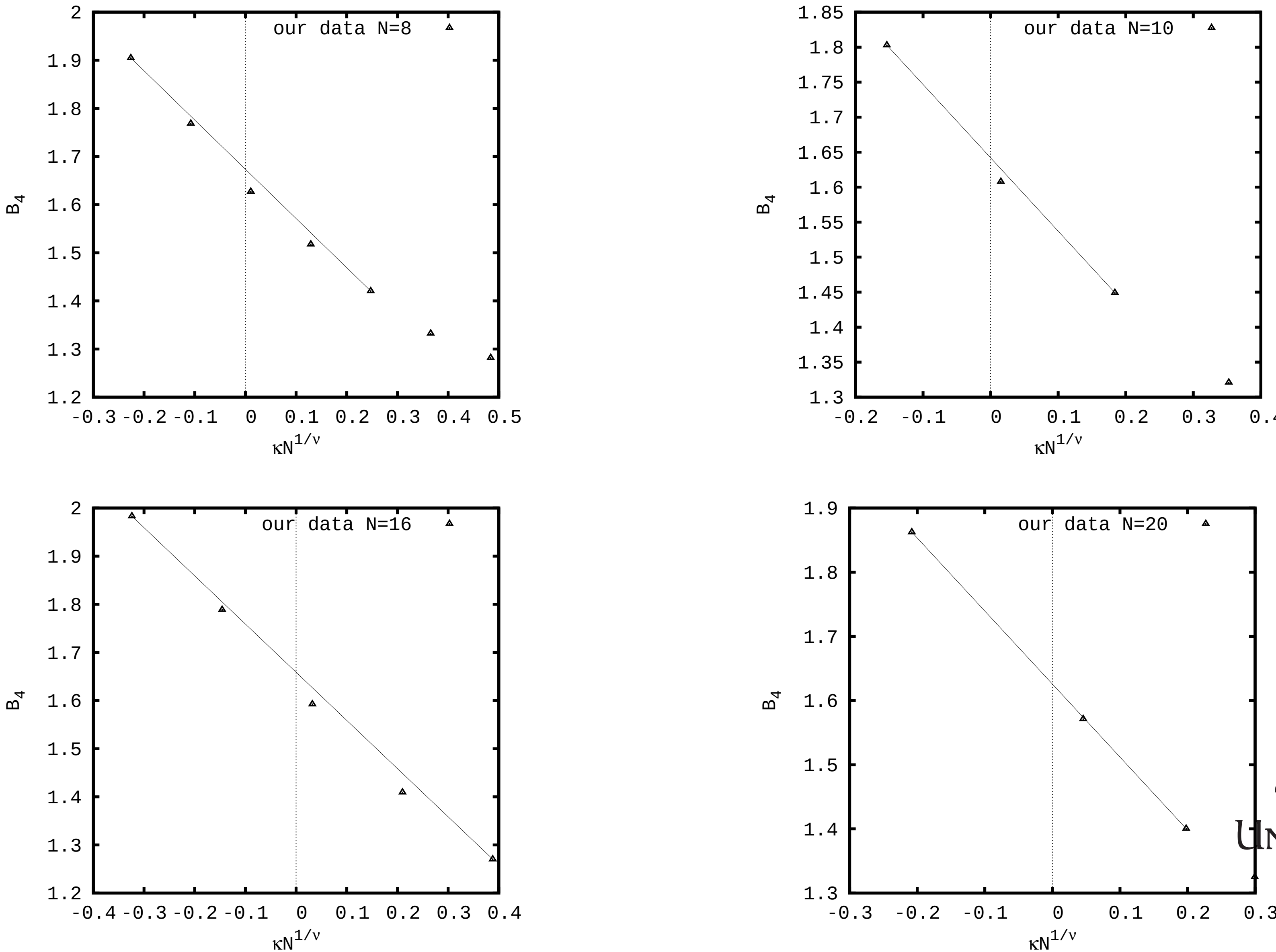
$$|\beta - \bar{\beta}_c| < \epsilon (f_1/f_2) \bar{\beta}_c N_\sigma^{-1/\nu} \ . \tag{1}$$

Y. Meurice arXiv: 0712.1190

$$\begin{aligned} f_1 &\simeq (y_1 - y_2)/2x \\ f_2 &\simeq -\Delta_0/x^2 \end{aligned}$$

THE UNIVERSITY OF IOWA

Determination of FSS Region

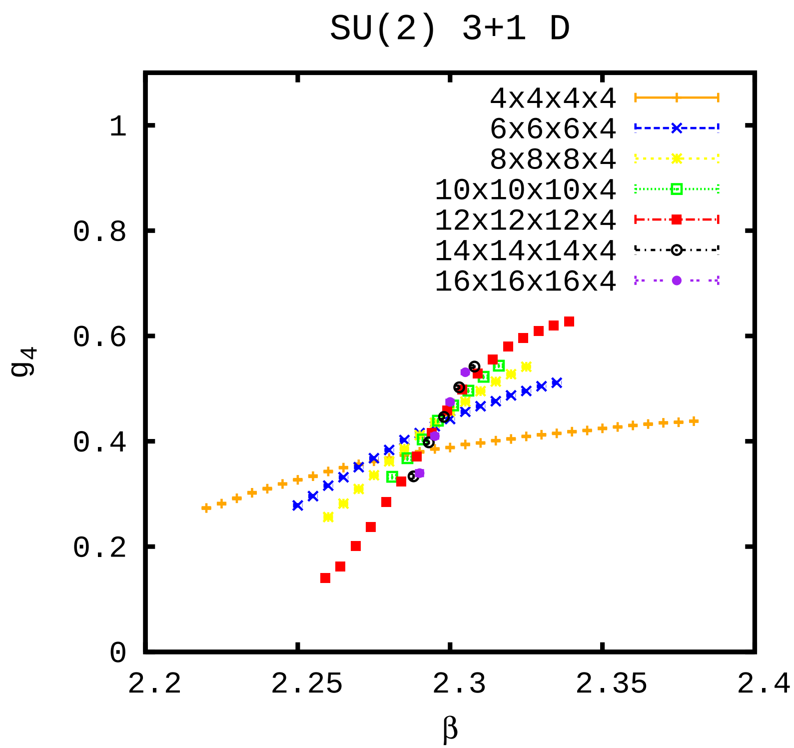


Determination of FSS Region

N_σ	f_1	f_2	f_1/f_2
8	-0.978613	-0.580271	1.68648
10	-0.961446	-0.537762	1.78787
16	-0.918545	-0.227324	4.04068
20	-1.166990	-0.061987	18.8262

Table 1: f_1,f_2 and f_1/f_2 for different volume(N_σ).

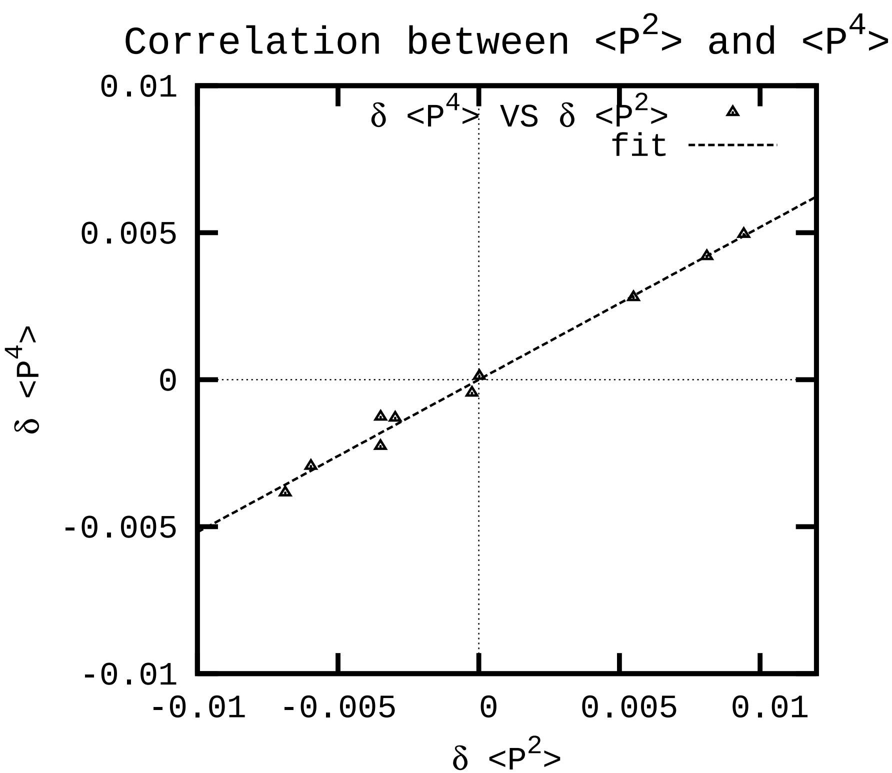
Critical Temperature



$$g_4(\beta, N_\sigma) = g_4(\beta_c, \infty) + f_1 \kappa N_\sigma^{1/\nu} + f_2 \kappa^2 N_\sigma^{2/\nu} + \cdots$$

Error Analysis

$$g_4 = 1 - \frac{\langle P^4 \rangle}{3 \langle P^2 \rangle^2}$$



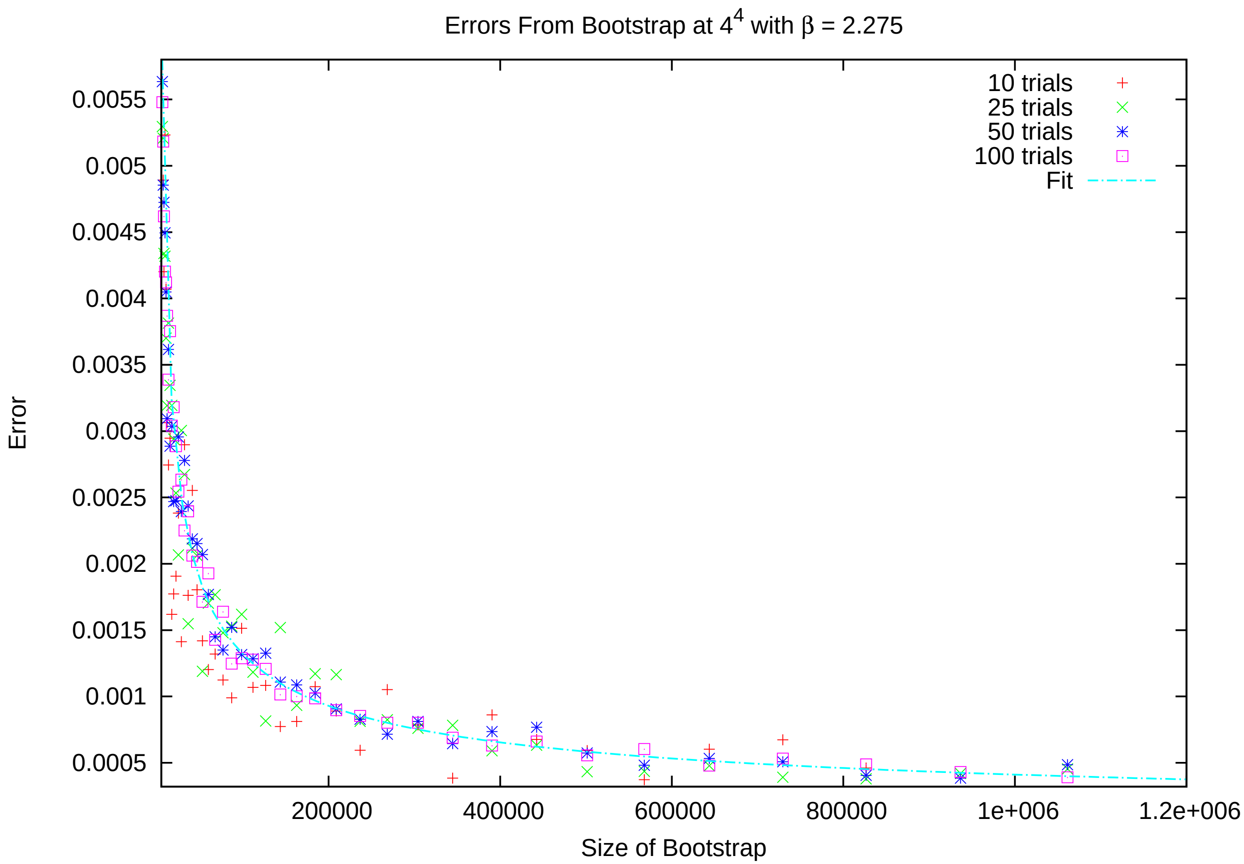
THE UNIVERSITY OF IOWA

Error Analysis

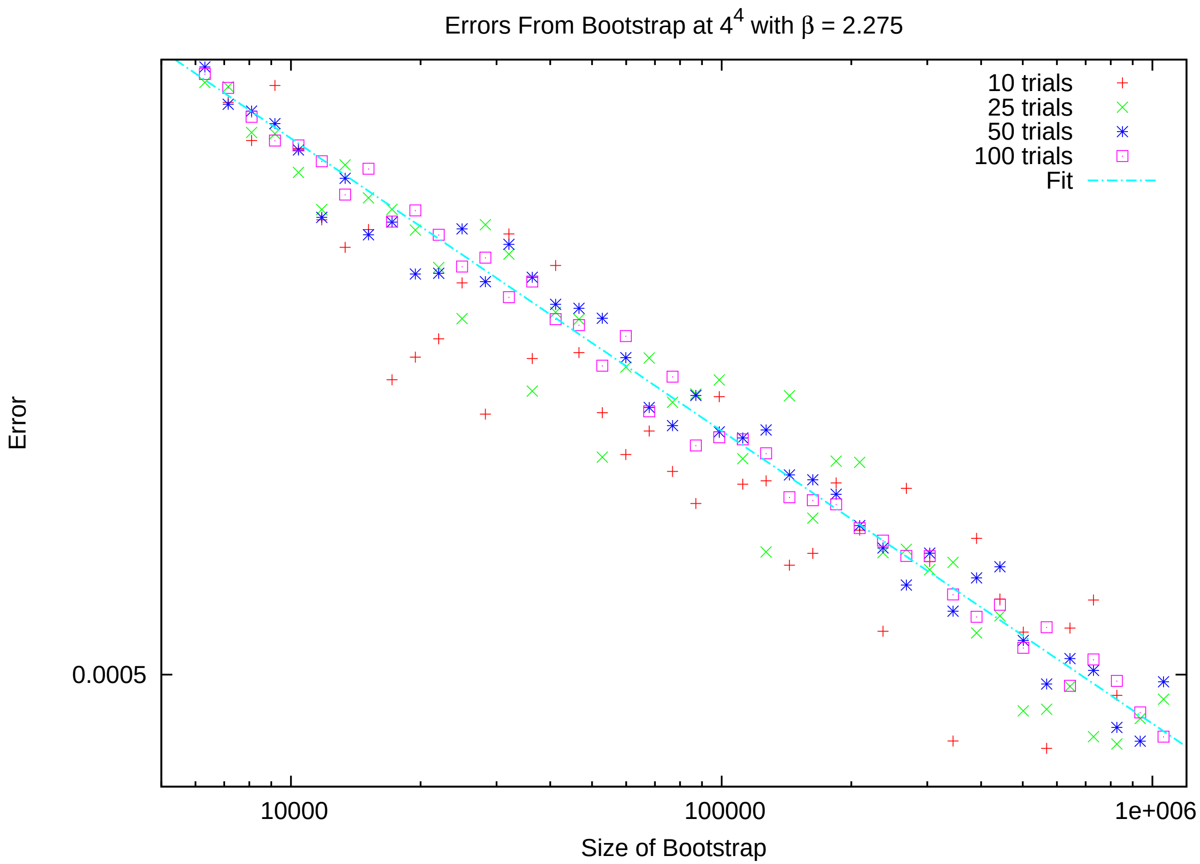
- $< P^4 >$ and $< P^2 >^2$ are highly correlated.
- Error Propagation overestimates the error by a factor of 10.
- Resampling method: Bootstrap and Jackknife.

THE UNIVERSITY OF IOWA

Error Analysis



Error Analysis



The error of the bootstrap scales with $N^{1/2}$.

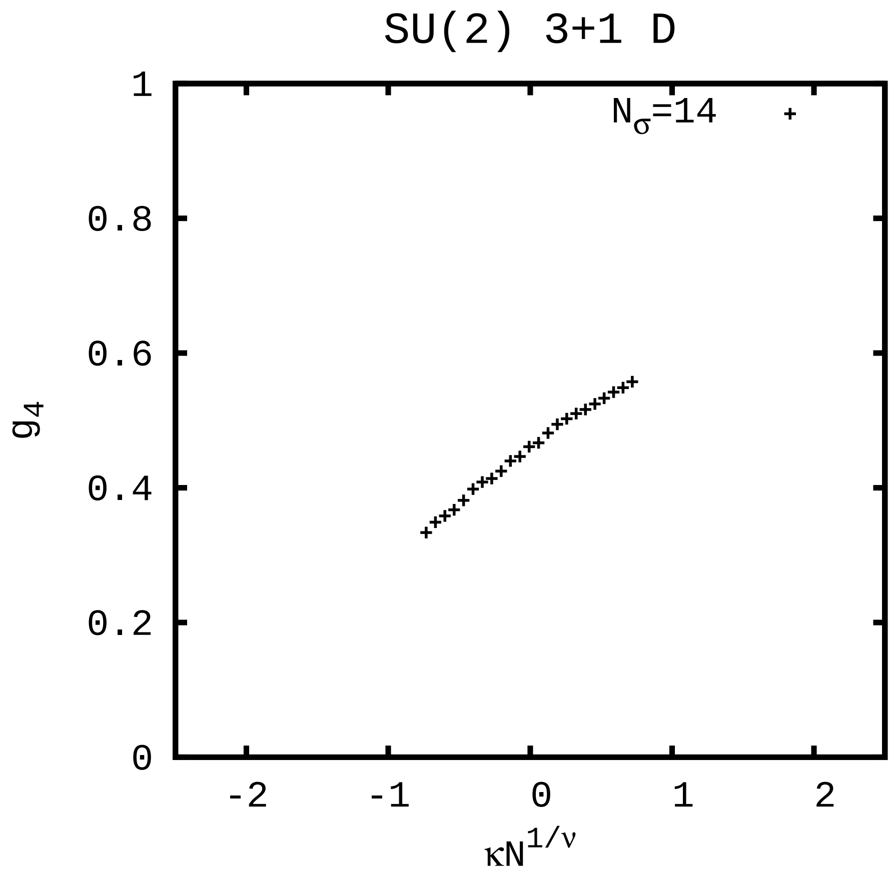
Critical Exponent

$$g_4(\beta, N_\sigma) = g_4(\beta_c, \infty) + f_1 \kappa N_\sigma^{1/\nu} + f_2 \kappa^2 N_\sigma^{2/\nu} + (c_0 + c_1 \kappa N_\sigma^{1/\nu}) N_\sigma^{-\omega} + \cdots$$

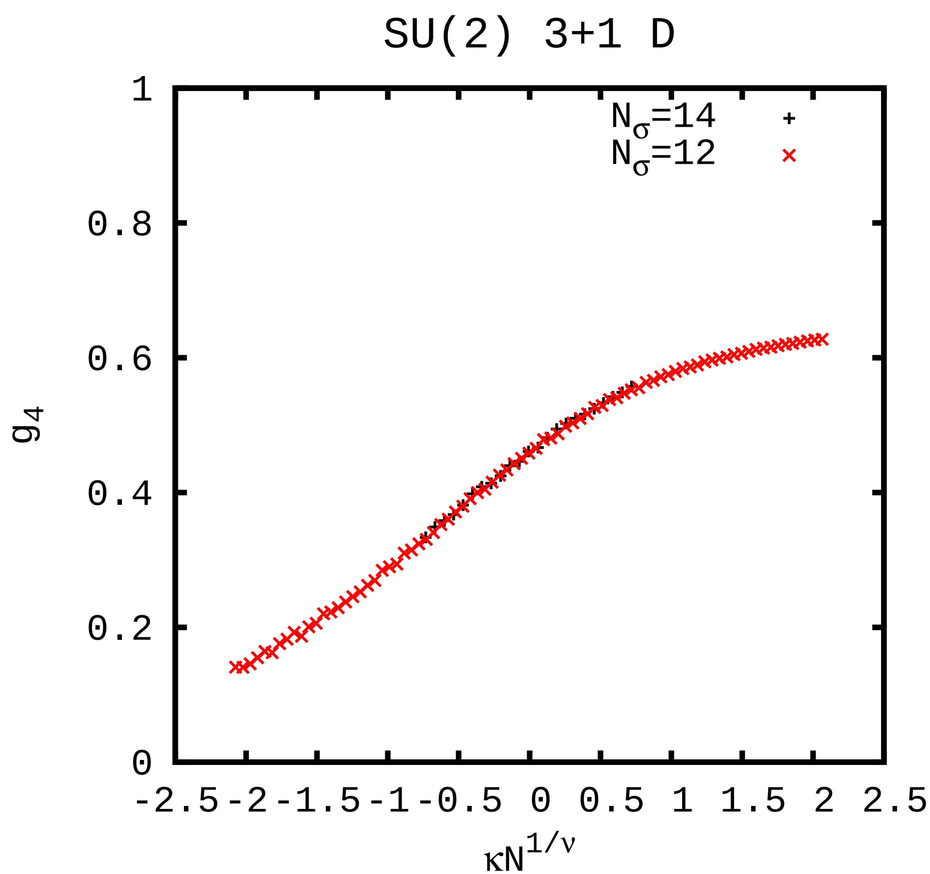
Critical Exponent

$$g_4(\beta, N_\sigma) = g_4(\beta_c, \infty) + f_1 \kappa N_\sigma^{1/\nu} + f_2 \kappa^2 N_\sigma^{2/\nu} + (c_0 + c_1 \kappa N_\sigma^{1/\nu}) N_\sigma^{-\omega} + \cdot$$

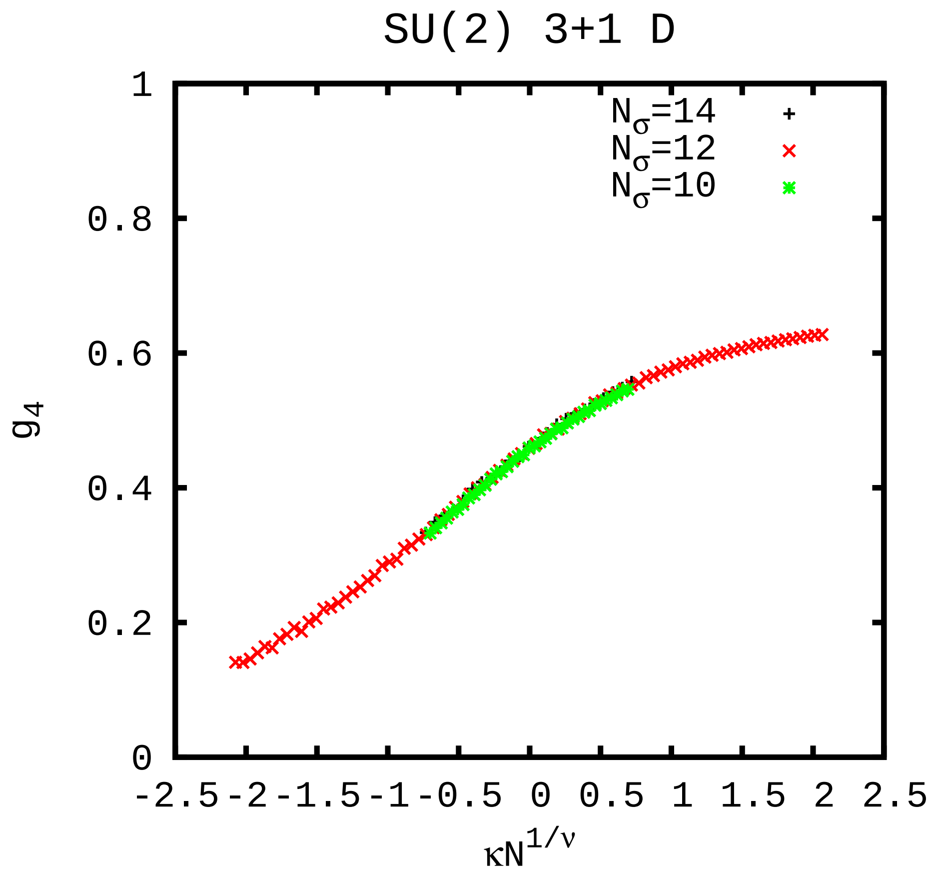
Critical Exponent



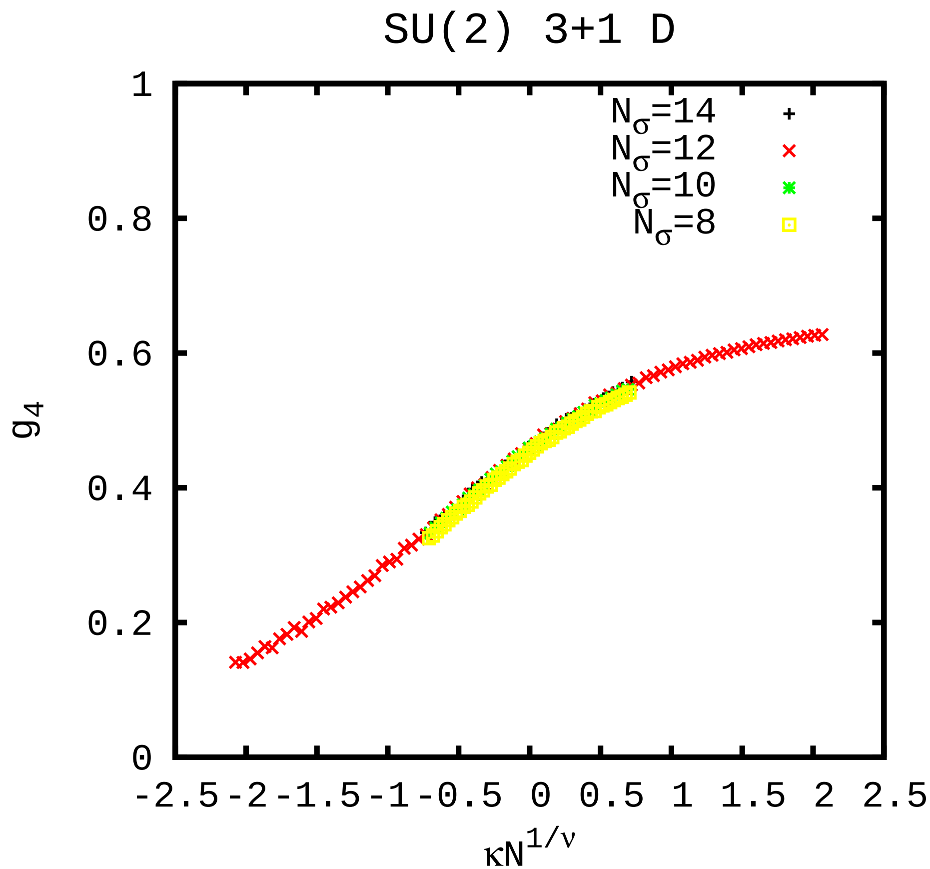
Critical Exponent



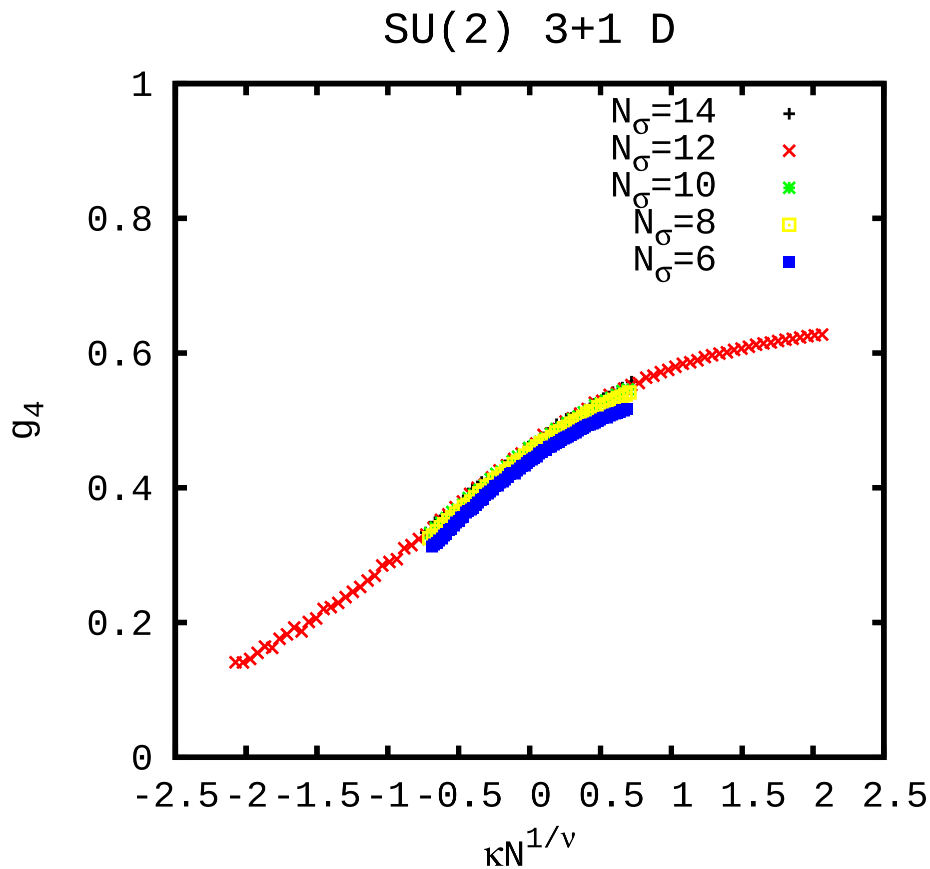
Critical Exponent



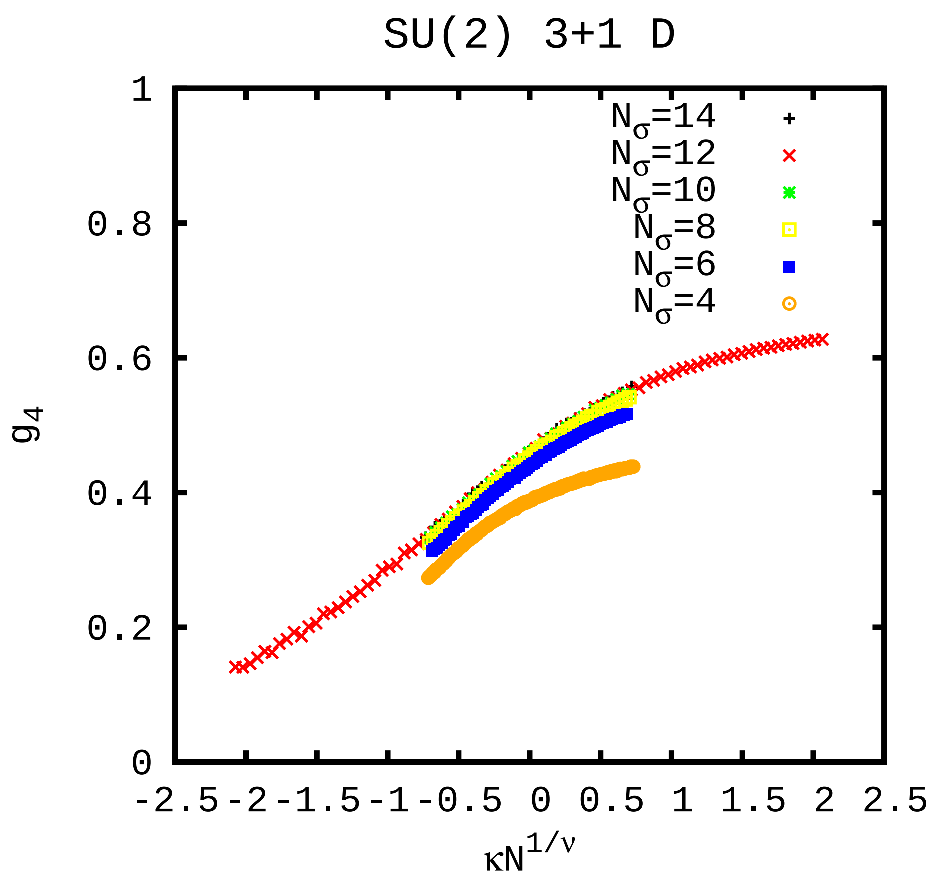
Critical Exponent



Critical Exponent



Critical Exponent



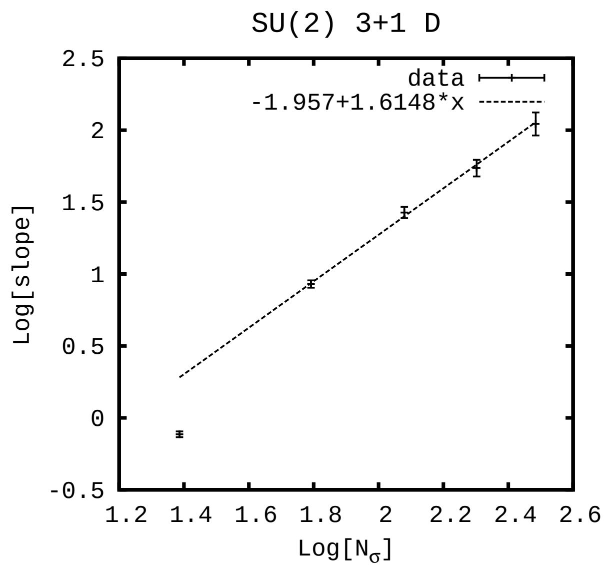
Critical Exponent

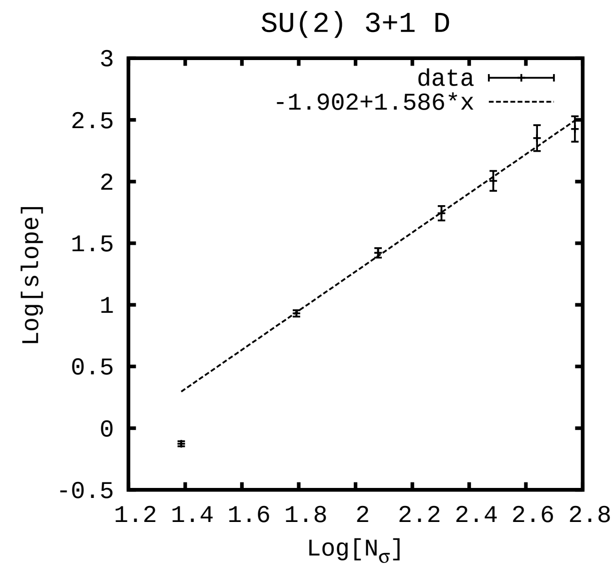
$$g_4(\beta, N_\sigma) = g_4(\beta_c, \infty) + f_1 \kappa N_\sigma^{1/\nu} + f_2 \kappa^2 N_\sigma^{2/\nu} + (c_0 + c_1 \kappa N_\sigma^{1/\nu}) N_\sigma^{-\omega} + \cdots$$

Critical Exponent

$$g_4(\beta, N_\sigma) = g_4(\beta_c, \infty) + \frac{f_1}{\beta_c} N_\sigma^{1/\nu} \beta + \cdots$$

Critical Exponent





Conclusion and Perspective

- 3+1 d SU(2) Gauge Model and 3-d Ising model belong to the same universality class.
- Error analysis of the data: Bootstrap and Jackknife.
- Reduce $1/\omega$ effect and get more accurate ν.
- Extract the critical exponent ω from small volume data.

THE UNIVERSITY OF IOWA